EXPERIMENTS IN IMPACT EXCITATION

BY

GEORGE WILLIAM NASMYTH

A THESIS

PRESENTED TO THE FACULTY OF THE GRADUATE SCHOOL OF CORNELL UNIVERSITY FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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BY GEORGE W. NASMYTH.

I. THE CHARACTERISTICS OF SHORT ARCS BETWEEN METAL ELECTRODES.

§ 1. *The Development of Impact Excitation and the Short Spark Generators.*

Impact Excitation.—The year 1906 marks the beginning of the history of "impact excitation" (Stosserregung) as a method of producing high frequency electric oscillations of considerable power which are only slightly damped. In October of that year Wien[1] published a brief account of his experiments "On the Intensity of the Two Waves in Coupled Transmitters," in which he found that for small spark lengths of about 0.15 mm., a third maximum appears between the maxima of the two waves usually met with in coupled circuits. This third maximum is higher than the other two, and the shape of the curve indicates that the damping of the corresponding oscillations must be very small. Its frequency corresponds to the natural frequency of the antenna, or secondary of the coupled circuits, whereas the frequencies of the two ordinary component waves are respectively greater and less than this natural frequency. According to the theory of impact excitation, the appearance of the third wave is due to the rapid recovery of the resistance of short gaps, so that the oscillations in the primary circuit are extinguished after they have given up the greater part of their energy to the secondary circuit. The oscillations in the secondary then continue as if it were an isolated circuit, with its own natural frequency and damping factor. The electric oscillations excited by "impact" correspond, therefore, to the vibrations of a pendulum which has been started by a blow, while the ordinary

[1]Max Wien, "Ueber die Intensitaet der beiden Schwingungen eines gekoppelten Senders," Verhandlung der Deutschen Physikalischen Gesellschaft, Vol. 8, 20, pp. 486–9, Oct. 30, 1906. See also Science Abstracts, Vol. 9, p. 633, no. 2076, 1906.

oscillations correspond to the vibrations of one of two coupled pendulums.

Lepel's Generator.—In the same year Lepel[1] began his work on the discharge between metal electrodes, and in November and December, 1906, invented a simple apparatus, using large iron electrodes separated by a film of oil, or Venetian turpentine. As early as 1890 and 1892 Elihu Thomson had succeeded in producing a type of discharge intermediate between the spark and the arc, using metal electrodes, but was unable to obtain either constancy of action or a sufficiently powerful oscillatory current for practical purposes. These difficulties were overcome by the two essential features in Lepel's device, the parallelism of the surfaces of the electrodes, thus making it possible for the discharge to take place at different points of the surface without altering the length of the path, and by the exclusion of air. Lepel later replaced the oil between the plates by mica, which was found unsuitable because it became conducting when heated, and then by one or two sheets of paper, which was found to answer every purpose. The iron electrodes were replaced by copper, either electrolytically pure or silver plated, and still later the negative electrode, when direct current was used, was made of brass or an especially dense bronze. By September, 1907, Lepel's apparatus had been so perfected that he was able to transmit messages and musical tones from Slough to Scheveningen, 160 miles, using an E.M.F. of 440 volts and 1.5 amperes of direct current.

Von Arco's "Quenched Spark."—The first public announcement of any of the short spark methods of generating oscillations was made by von Arco[2] in April, 1909. The Telefunken Company had experimented with Lepel's apparatus early in 1907, but finding it unsatisfactory in its form at that time, proceeded to develop its own system. The gap at present used by the Telefunken Company

[1]The best descriptions of Baron von Lepel's system and apparatus are to be found in The Electrician (London), Vol. 63, pp. 142, 157, 174, 345, 376, 1909; Vol. 64, pp. 153, 386, 1909–10. Abstracts from his German patent no. 24,757, filed August 20, 1907, are given in Vol. 63, p. 374, 1909.

[2]The Electrician, Vol. 63, p. 89, "A New System of Wireless Telegraphy used by the Telefunken Company," by Count Arco, April 30, 1909. See also pp. 332, 370, 461 and 720. For the relation and comparison of the Lepel and Arco systems, see also Vol. 63, pp. 228, 325, 617, 661; Vol. 64, pp. 726, 768 and 947.

differs from Lepel's gap in the use of a mica ring instead of paper, a groove being cut in the electrodes at the inner edge of the mica to prevent the discharge occurring at that place. The distance between the electrodes is about 0.25 mm., and several gaps are used in series to increase the power. The electrodes are provided with flanges to assist in radiating the heat. The secondary or antenna circuit is closely coupled to the primary so that the energy of the spark discharge is quickly absorbed. The company has given the name "quenched spark" (Loeschfunken) to this form of excitation, which is identical in principle with Wien's Stosserregung. Using an alternating current at a frequency of about 2,000 cycles the sparks may be made to follow each other so equally and regularly that they give out a clear musical tone, and this system is known as the "singing spark" (toenende Funken) system.

The practical and technical advantages of the new system over the older spark methods as well as over the undamped arc oscillations have been given by von Arco under twelve headings. Since most of these advantages are common to the other short spark systems, they are briefly summarized below:

1. Considerably smaller antenna may be used. This is due to the speed with which the sparks follow each other, so that greater power may be transmitted from the antenna at a smaller maximum voltage, and to the excellent characteristics of the short spark as a generator of short waves.

2. High overall efficiency. It is claimed that with the singing spark from 50 per cent. to 75 per cent. of the machine output can be changed into energy at the antenna, as contrasted with 20 per cent. for the older spark system and 10 per cent. for the arc lamp generators.

3. Adaptability for high power sending stations. No such limitations in the wave-length or difficulties in keeping the oscillations constant as arise in older arc methods present themselves.

4. Noiseless operation.

5. One wave is transmitted instead of two.

6. Small damping of wave sent out, from 0.08 to 0.025.

7. Constancy of the oscillations.

8. Large range of frequencies available, and the possibilities

of using overtones whose frequencies are four, five or even six times the fundamental.

9. Military advantages of using short waves which are absorbed before they travel a longer distance than is intended.

10. Sharp tuning due to constancy of oscillations and small damping of radiated wave. The freedom from disturbances is from 2 per cent. to 5 per cent., as contrasted with 20 per cent. to 30 per cent., which may in exceptional cases be reduced to 5 per cent. or 6 per cent., in arc lamp practice.

11. Elimination of atmospheric disturbances and individuality of musical tones.

12. Acoustic tuning of receivers which enables relays to be used for calling, etc.

Peukert's Rotating Disk Generator.—Recently Professor W. Peukert[1] has announced the invention of a short spark generator in 1906, for which a patent application was filed February, 1907, prior to either the Lepel or Telefunken Company applications. Peukert's apparatus consists essentially of two carefully surfaced metal disks held together by a spring pressing against one of them, with a filling of oil between. One disk is rotated at a constant speed of about 800 r.p.m., and oil is fed regularly at the center of the fixed disk and distributes itself in a uniform capillary layer between the plates. The frequency of the discharges depends upon the constants of the circuits, but in one test it was found to be 48,900 per second, as measured by the radiation to a revolving photographic plate from a needle point connected to a circuit coupled to the secondary. The gap efficiency was found to be as high as 60.2 per cent. in one test, with an applied E.M.F. of 615 volts; but the overall efficiency was not given. The generator may be made to give a definite pulsation to the oscillatory energy, thus producing a musical note in the telephone receiver, by dividing the disks into sectors, each alternate one consisting of insulating material. In this form it resembles Marconi's studded disk discharger. In its origin,

[1]The Electrician, Vol. 64, p. 361. See also p. 550 for an abstract of a dissertation presented to the Brunswick Technische Hochschule by Dr. A. Wasmus on Peukert's High Frequency Generator for Wireless Telegraphy on the Quenched Spark Method, and Peukert's English patent 4762 (1909).

Peukert's generator goes back to the results of Steidles' work[1] on contact between a spring and a rotating oil-covered disk against which it presses.

These three short spark generators constitute the chief means at present available for producing high frequency oscillations of the impact excitation type. The principle has opened up a new region and given the investigator of electric waves and oscillations a new tool, and its practical applications are so important in radio-telegraphy that new forms of generators and improved modifications of these present forms will undoubtedly result. Indeed, Wien[2] has recently announced that he has been able to obtain impact excitation effects with a spark gap of the usual length by including a vacuum tube in the condenser circuit. A wide tube having a pressure of 0.5 to 0.1 mm. of mercury, and two aluminium electrodes 2 to 3 cm. apart, was used. With a degree of coupling from 6 to 12 per cent. only one wave was obtained, the primary oscillations being damped out in a short time by the rapid recovery of resistance of the vacuum tube.

In principle the three spark generators have much in common, and experiments and theory dealing with any one will have a measure of value for each of the others. The author's experiments have been made on the Lepel generator. This is the simplest of the three, since it uses direct current instead of alternating, as in the case of the Telefunken generator, and it has no moving parts, as in the case of Peukert's apparatus.

Arc or Spark.—The discharge between the electrodes of all three generators is intermediate in character between the ordinary arc and spark discharges, and in the absence of exact definitions for the two terms, it is difficult to decide in which class they should be placed. From the arc point of view, Lepel's generator is a d.c. arc between metal electrodes, the Telefunken arrangement is an a.c. arc, and Peukert's generator is an arc between relatively rotating electrodes. But in general when the capacity circuit is connected the discharge of all three generators becomes discontinuous, and

[1]Elektrotechnische Zeitschrift, Vol. 26, p. 679, 1905.

[2]Max Wien, Ueber ein neue Form der Stosserregung elektrischer Schwingungen; Physikalische Zeitschrift, Vol. 11, p. 76, February 1, 1910.

corresponds to a spark gap. This condition also arises in the Poulson arc, although to a less extent. Von Arco has suggested the following definitions: "By arc excitation one might understand that method of producing oscillations by which the rate of condenser discharges is equal to or about the frequency of the oscillations; by spark excitations, that method by which this rate is considerably less." This definition makes the type of discharge depend upon the constants of the circuit, and the same generator might be an arc or a spark gap under different conditions. The author's experiments on the Lepel generator have demonstrated that in many of its general characteristics it bears a marked similarity to the Poulson arc, and for the present it will be called an arc whenever the applied voltage would be sufficient to maintain the current through the gap if the condenser circuit were disconnected.

§ 2. *Three Kinds of Oscillations.*

Besides the familiar damped discharge of the ordinary spark gap, three distinct types of arc oscillations have been recognized quite generally by investigators[1] in this field. For convenience the author has named these types after the experimenters who first produced them:

1. *Duddell Oscillations.*—The characteristic of these oscillations is that the maximum value of the condenser current is less than that of the direct current through the arc, so that the arc remains lighted. These are the true "singing arc" or "musical arc" oscillations, since the frequency is always within the audible range. The efficiency of these oscillations falls off rapidly as the frequency is increased, the upper limit being about 10,000. Although these oscillations are not available for radio-telegraphy on account of their low frequency, their discovery by Duddell[2] in 1900 marks the beginning of the development of undamped oscillations, and led to the discovery of the second type.

2. *Poulsen Oscillations.*—In these oscillations the maximum value of the condenser current is greater than that of the direct

[1]See K. Vollmer, Jahrbuch der Drahtlosen Telegraphie und Telephonie, Vol. 3, p. 119, December, 1909.

[2]W. Duddell, Journ. Inst. Elec. Engineers, Vol. 30, p. 232, 1900.

current through the arc, so that the arc is extinguished and relighted at each oscillation. In the Poulsen[1] arc, which is the Duddell arc immersed in an atmosphere of hydrogen and having one electrode of copper, water-cooled, oscillations having a frequency of a million per second or more may be obtained with good efficiency. These oscillations may also be produced by connecting several arcs in series, as Simon has demonstrated. Oscillations of this second or Poulsen type have been used extensively in radio-telephony. Since these oscillations are usually above the audible range, they can be called "singing arc oscillations" only by analogy.

3. *Wien Oscillations.*—Here the maximum value of the condenser current is so great that the arc is not only extinguished, but is relighted in the opposite sense. Then follows a series of damped oscillations through the arc of the well-known kind, until the potential difference is no longer sufficient to relight the arc. The condenser is then charged by the direct current until the arc resistance breaks down again, and another train of damped oscillations appears and disappears. These oscillations are easily produced if both electrodes are of metal and the arc length is short. These are the oscillations usually known as "Stosserregung" or "impact excitation" oscillations.[2]

It is with this third type, or Wien oscillations, that this investigation is chiefly concerned. The Duddell and Poulsen oscillations have been the subject of many interesting and important researches, but up to the present very few details of quantitative experiments have been published on the subject of the Wien oscillations or the characteristics of the new type of generators.

§ 3. *Arrangement of Experimental Apparatus.*

Besides the general working characteristics of an impact excitation generator, three characteristics are of especial interest: the form of the voltage current curves, on account of their importance in the theory of the action of apparatus; the intensity of the oscillations as a function of the arc current and the constants of the circuits, on account of its relation to the efficiency of the generator;

[1] V. Poulsen, British Patent Specification 15599 of 1903. See also Science Abstracts, Vol. 8A, p. 521.

[2] Max Wien, *l. c.*

and the influence of the discharger upon the frequency of the oscillations which it produces. In the author's experiments, on the Lepel arc, the arrangement of apparatus, as shown in Fig. 1, was designed with special reference to these three characteristics.

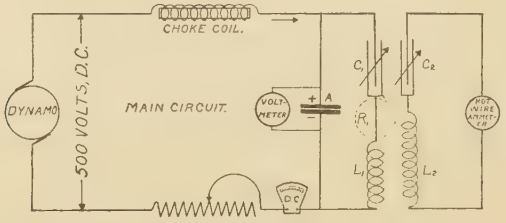

Fig. 1.

Diagram of connections.

The source of E.M.F. was a 500-volt shunt wound dynamo, direct connected to a two-phase, 900 r.p.m. induction motor. At first a Tirrell regulator was used on the dynamo, but later it was disconnected for fear that the pulsating current which resulted might introduce irregularities into the experiments. Without the regulator the voltage of the generator was found to be very steady over short intervals of a few minutes, although over intervals of a half hour or more it was subject to slow drifts of $\pm$ 20 volts. A large water rheostat made from an old oil barrel, together with two Cutler-Hammer rheostats arranged for connection in series or parallel, each having a maximum resistance of 200 ohms and a current capacity of 1.5 to 2.5 amperes, constituted the resistance in the circuit. The choke coils were two solenoids 47.5 cm. long, wound with seven layers of 192 turns each of double cotton covered copper wire 2.05 mm. in diameter. The inside diameter was 8.2 cm. and the outside 11.4 cm. The coefficient of self induction, measured by comparison with an Ayrton-Perry standard, was 0.0296 henry for each coil. In the condenser circuit, a variable number of Leyden jars, having a capacity of 0.00188 microfarad each, were connected in parallel. The inductance in the primary circuit was also varied but for most of the experiments a solenoid 20.55 cm. long, having a mean diameter of 12.85 cm. and consisting of 15 turns of stranded wire insulated and wound on a glass jar, was used. The diameter

of the wire, which was made up of 36 strands, was 0.65 cm.; the coefficient of self induction of this solenoid was 14,300 cm., and the resistance of the condenser circuit with this solenoid connected and the capacity short circuited, was only 0.15 ohms, as measured by a Wheatstone bridge.

A Weston d.c. voltmeter of the d'Arsonval galvanometer type, with a 600-volt and 150-volt scales, was connected across the arc, and a Weston millivoltmeter, with a 5-ampere shunt, was connected in series with the main circuit, as shown. The intensity of the oscillations was measured by an unshunted hot wire ammeter containing no iron and giving a maximum deflection at 7 amperes of direct current. The position of the ammeter when used for measuring the oscillations is shown by the dotted circle in the primary circuit. The expanding wire was of copper, 0.22 mm. in diameter and 9 cm. long, and the resistance of the instrument is included in the 0.15 ohms mentioned above. The hot wire ammeter was calibrated before and after reading by short circuiting the capacity and causing a direct current to flow through the hot wire ammeter and the main circuit ammeter in series.

§ 4. *The Lepel Arc.*

After experimenting with various forms of the Lepel apparatus, it was found that with flat copper plates water cooling of the electrodes was unnecessary, as the radiation to the air was large, and the generator seemed to work better, if anything, as it became heated by the discharge.

The Electrodes.—The author's first experiments were made with brass and copper plates. The results obtained when either metal was used as cathode were fairly satisfactory, but the pitting of the brass plate, due possibly to the volatilization of the zinc, was found to be a source of great annoyance and to make the oscillations inconstant by changing the arc length. Two electrodes of sheet copper ¼ inch thick and 6 inches in diameter were then obtained. These were carefully surfaced in a lathe with a sharp tool set at an acute angle, and were then rubbed with sandpaper and oil. The connecting wires were at first soldered to the copper plates, but after they had been melted off two or three times, they were fastened

to screws set into the edges of the plates. These electrodes were found very satisfactory, and Fig. 2 is from a photograph showing the appearance of the electrodes and insulating paper after they had been used almost continuously for six hours. Although the paper was burned brown and the copper discolored by the heat they were working as well at the end of the run as at the beginning, and after the bluish deposit of carbon from the burnt paper had been rubbed off with emery cloth the electrodes were ready for another test.

Fig. 2.

Photograph showing appearance of copper plates and paper after a six-hour run.

The oscillations are more irregular when the surfaces of the electrodes have been rubbed with emery cloth than when oil and a sandpaper block is used. When the emery is used it is probable that little points remain sticking in the copper. On the other hand, the vapor of the oil (ordinary machine lubricating oil was used) seemed to have a marked effect in steadying the arc conditions.

Very good electrodes may be made from the hard rolled copper plate used in the photo-engraving processes. The surface of this copper is very smooth and flat, and it holds its shape well on account of its hardness. If this copper sheeting is used, it is unnecessary to turn the plates to a flat surface in a lathe, an operation requiring

some skill. The plates need only be rubbed with fine pumice stone and oil, and then with a piece of wood carbon and oil, and the surface is almost perfect. Contact with the connecting wires may be made by means of a piece of tinfoil held down by a small weight.

The Insulator.—Any kind of paper seems to work well as an insulator and as a source, when disintegrated by the discharge, of the hydrogen and carbon atmosphere which the arc requires for satisfactory working. For the sake of uniformity in conditions, W.S. & B. Paragon typewriter paper in legal size sheets was used in all the quantitative experiments. In thickness this paper is very closely 0.05 mm. per sheet and the arc length was varied by increments of this amount by using a varying number of thicknesses. Mica rings were tried without success, as the discharge occurred either at the edge of the mica, or, if this were prevented by cuting a sufficiently deep groove in the plates, the discharge took place through the mica instead of across the air gap. Since there is no hydrogen atmosphere present when mica is used, the potential difference required for sparking is higher. Moreover, the insulating properties of mica disappear in large measure with rise of temperature. Water cooling is probably essential for the successful use of mica, and in addition to the groove at the edge the plates should be planed off outside the groove, so that the distance between the plates through the mica is greater than the distance through air gap.

Starting the Arc.—If a potential difference of 500 volts is applied to the electrodes of the Lepel arc when they are separated by two or three sheets of paper having a small hole in the center, the discharge will not in general take place. If the insulation of the gap can be broken down, however, the discharge will continue, the average voltmeter reading across the arc being from 37 v. upwards, according to the current, arc length, etc. To start the arc various devices may be used, the most convenient one being to short circuit the arc and then suddenly break the short circuit, thus bringing the full pressure of the inductive kick of the choke coils to bear on the gap. To break the short circuit suddenly, the author has used the following simple form of oil switch. One electrode of the arc is connected to a brass binding post having a flat top. Insulating oil poured on

this will be held by surface tension to a depth of a millimeter or more. Then a piece of brass having a wooden handle and connected to the other electrode is touched to the top of the binding post through the oil, making a short circuit across the gap. The circuit is then broken in the oil, resulting in a high potential difference across the gap which is usually sufficient to start the discharge. Theory shows that the inductive kick is greatest when the resistance in the main circuit is least, *i. e.*, when a large current flowing through the short circuit is suddenly interrupted.

If the edge of the hole in the paper has been charred, by a match or some other means, the discharge will start as soon as the potential difference is applied. After the discharge has been stopped and the electrodes taken apart, it will usually start automatically if the same paper is used when the apparatus is put together again. If the paper is charred too much, either a short circuit will result, giving a voltmeter reading of from 20 to 40 volts, or an inactive stage of the arc sets in which does not produce oscillations although the voltmeter gives a steady deflection of about 325 volts and a current of two or more amperes flows through the arc. To start the oscillations again after this inactive stage has set in, either the plates must be rubbed clean or the charred part of the paper must be moved to a new place on the plates.

Arcs in Series and Parallel.—As in the case of the Poulsen arc the Lepel generator may be made to give increased power by connecting several arcs in series. By starting the arcs one at a time separately and then connecting them together in series, the author has been able to make as many as three arcs run on 500 volts impressed E.M.F. This was only possible at relatively large currents, however; at smaller currents the arcs would go out and it would be necessary to start from the beginning again. The power obtained in the oscillations with three arcs in series was considerably less than three times the power delivered by one arc.

A better way to obtain increased power at a given impressed E.M.F. is to connect two arcs in parallel, each with its own choke coil and resistance in series. If the two cathodes (or anodes) are connected together, and the capacity and inductance of the condenser circuit connected to the two anodes (or cathodes) the same

potential difference will be required as for one arc alone and the two arcs will add their characteristics, doubling the power available for the oscillations. The critical resistance in the condenser circuit at which oscillations can be obtained is equal to the sum of the absolute values of the negative slopes of the volts-amperes characteristic curves for the two arcs, and is therefore about double the resistance which can be used with a single arc.

The method of generating oscillations by connecting arcs in parallel in this way was discovered by the author in 1907 in attempting to apply the principle of Vreeland's mercury vapor magnetic field oscillator to the arc, and the advantages of the connection were set forth under the title "An Improved Form of the Duddell Singing Arc."[1] Later the attention of the author was called by Mr. Duddell to a lecture given in England before the Institution of Electrical Engineers in 1900, in which practically the same principle had been used. The author had overlooked the reference to the earlier work, and is glad to take this opportunity of acknowledging the priority of Duddell in discovering the method of generating oscillations by connecting arcs in parallel.

§ 5. *The Volts-Ampères Characteristic Curves.*

The form of the characteristic curves between the voltage as measured by a d.c. voltmeter across the arc and the current through the arc is of considerable theoretical and practical importance. According to the theory of the Duddell oscillations, in order that oscillations may be produced, the curve must have a negative slope and its value, $-dV/dA$, must be greater than the resistance of the shunt circuit. From its similarity to the Duddell and Poulsen arcs, it might be inferred that a similar law would hold for the Lepel arc. Moreover, the shape of the characteristic curve has an important bearing upon the question of the variation of frequency in the oscillations produced by the arc. One of the first problems in the author's experiments, therefore, was to determine the form of these curves.

The characteristic curve for an arc length of 0.15 mm. when the arc is generating oscillations is shown in Fig. 3. Three sheets

[1] PHYS. REV., Vol. 28, p. 459, 1909.

of paper were used, and the capacity and inductance of the oscillation circuit are given. The points plotted are from tests made on two successive days, each extending over the whole range of the curve. For currents much larger than 4 amperes the arc be-

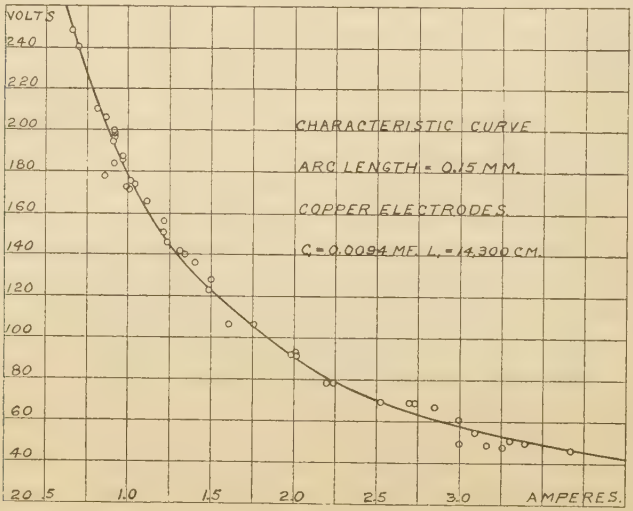

Fig. 3.

Characteristic volts-amperes curve for the Lepel arc with oscillations.

comes inactive and the oscillations cease, probably on account of the flatness of the curve for large currents. For currents less than 0.7 ampere it is difficult to obtain steady conditions on account of the steepness of the curve. Fig. 3 shows therefore the range over which steady oscillations can be obtained with an arc length of 0.15 mm.

The readings of the d.c. instruments represented in Fig. 3 are of course average or integrated values. Theory shows that with the large inductance of the choke coils in the main circuit the current varies but little during the charge and discharge of the condensers. The fluctuations of the voltage may be very great, however, and the voltmeter reading, expressed mathematically, is

$$V = \frac{I}{t}\int_0^t v\,dt,$$

where v is the instantaneous voltage and is integrated over the complete cycle of charge and discharge. To obtain the complete history of the arc current and voltage during the cycle it would be necessary to employ an oscillograph.

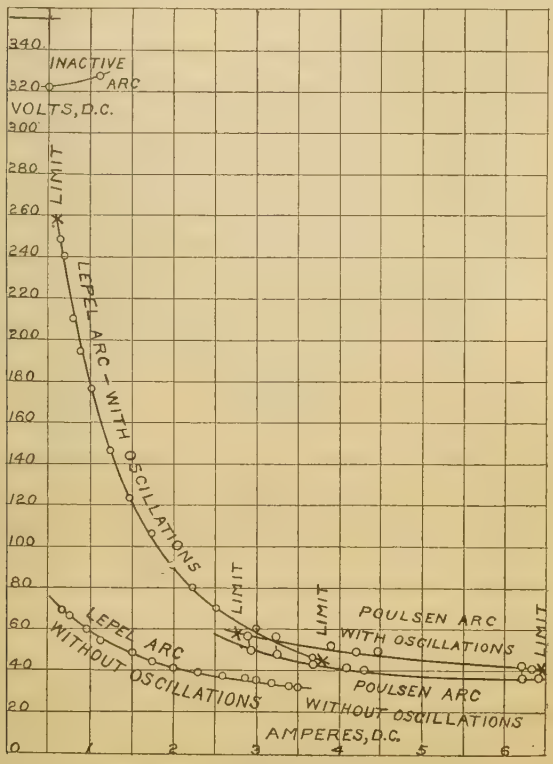

Fig. 4.

Comparison of Lepel and Poulsen arc characteristics.

The relation of the Lepel arc characteristics to those of the Poulsen arc are shown in Fig. 4. The upper and lower curves for the Poulsen arc are taken from an earlier paper[1] by the author and represent the characteristics for an arc between a water-cooled

[1]The Frequency of the Singing Arc, PHYS. REV., Vol. 27, p. 126, 1908.

copper anode and a graphite cathode in an atmosphere of illuminating gas when the switch in the oscillation circuit is open and closed, respectively. The points for the corresponding curve for the Lepel arc "with oscillations" are taken from Fig. 3; the lower curve for the Lepel arc gives the corresponding characteristic when the condenser circuit is disconnected.

One reason for the greater effectiveness of the Lepel arc is due to the steepness of the characteristic in the region of small currents in which it operates. Even at larger currents, however, where the two generators can be directly compared, the Lepel characteristic seems to be much steeper than that for the Poulsen arc. The separation of the two curves for the Lepel arc is also seen to be much greater than that for the Poulsen arc. In experiments with the Poulsen arc, the author has found that the oscillations cannot be started unless the resistance of the condenser circuit is less than the smaller of the negative slopes of the characteristics, but that when once started the resistance can be increased until it is but slightly less than the negative slope of the steeper characteristic. If the same conditions hold for the Lepel arc, the resistance would still have to be quite small in order to have the oscillations start, on account of the small value of the slope of the lower characteristic, but once started, it should be possible to increase the resistance through a large range before it becomes equal to the negative slope of the characteristic "with oscillations."

The limits of the current between which oscillations could be obtained in the author's experiments are indicated by crosses in Fig. 3. At small currents the characteristic of the Lepel arc becomes so steep that the oscillations are very irregular, and although oscillations have been obtained at currents less than 0.5 amperes, they were transient or very unsteady. At larger currents the characteristic becomes too flat, but by decreasing the resistance of the shunt circuit still further oscillations could be obtained with currents larger than 4 amperes. The product of the difference of the voltages of the two characteristics by the corresponding amperes may be taken as a measure of the power converted into oscillations. Since the characteristic "without oscillations" varies but little for the different arc lengths, a comparison of the characteristics "with

oscillations" will furnish a means of finding the arc length and current which will transform most power into oscillations and give the greater efficiency. It will be found from the three following figures that with with the author's apparatus the maximum power and efficiency are obtained at an arc length of 0.2 mm. and with a current between 1.0 and 1.5 amperes.

An inactive state of the arc which results sometimes when it is extinguished and started again, or when the paper is charred too much, is represented by the short curve between 320 and 330 volts. When this inactive stage sets in, the plates must be taken apart and the surfaces cleaned before steady oscillations can be again obtained. A condition which sometimes exists when the oscillation circuit is opened is represented by the horizontal line at 356 volts. This

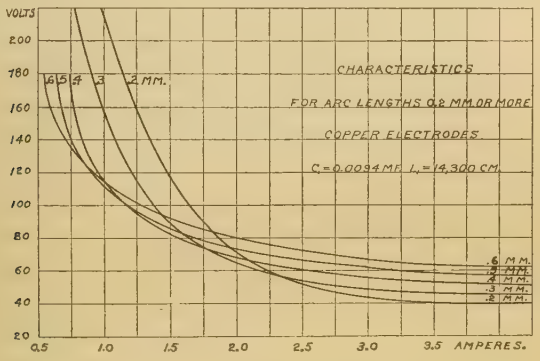

Fig. 5.

corresponds to the minimum sparking potential between copper electrodes, and seems to be accompanied by a small leakage current. When the oscillation circuit is again connected the oscillations do not always start, but the voltage remains at 356 no matter how small a resistance is placed in series with the arc.

In attempting to find the change in the characteristic curve when the arc length was varied, apparently discordant results were obtained. For large currents the voltage increased with increasing arc length, as in the case of the Poulsen arc, but for small currents the voltage sometimes increased and sometimes decreased with

increase of arc length. It was then decided to make a systematic exploration of the whole characteristic field, taking a hundred or more observations at each arc length under different conditions and at different times, and drawing a smooth curve through the observations, as in Fig. 3. The results of these experiments are shown in Figs. 5 and 6, representing more than a thousand single observations. The observations for each arc length were drawn on a separate sheet, and when a transparent sheet of cross-section paper was placed over these and the curves traced one after another, the cause of the apparent discrepancies became clear.

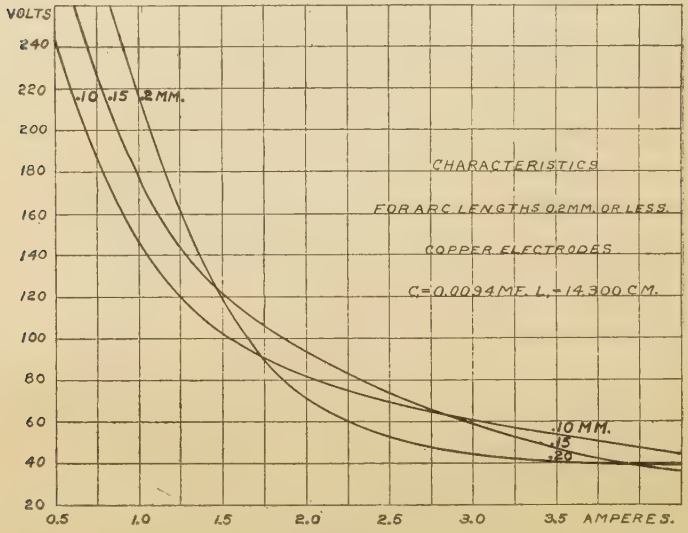

Fig. 6.

Lepel arc characteristics for different arc-lengths.

Although at large currents the curves tend to lie one above the other in the order of the corresponding arc length, at low currents the case is far different. To avoid confusion the characteristic curves have been plotted in two groups, the common characteristic for the 0.2 mm. arc length serving as a standard of reference. It is evident from these curves that for small currents (*e.g.*, one ampere) as the arc length is increased from 0.1 to 0.2 mm., the voltage increases ;

then as the arc length is increased to 0.5 mm. the voltage falls off rapidly at first, then more slowly, and begins to rise again as the length is increased to 0.6 mm. If cross-sections of the curves are taken in this way at constant currents, and the potential difference across the arc plotted against the arc length, the family of curves shown in Fig. 7 is obtained.

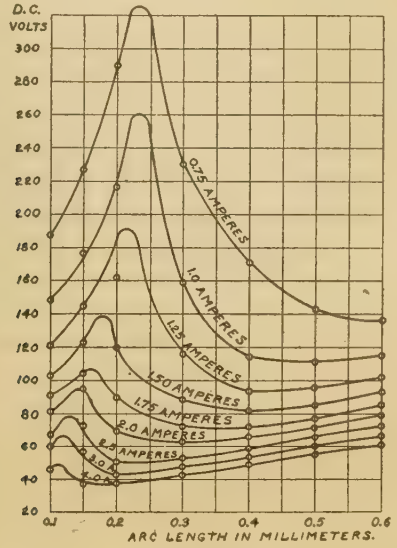

Fig. 7.

P.d. across arc for constant current and varying arc length.

The form of the curves suggested a resonance phenomenon, and tests were at once made to determine whether the effect was a true one corresponding to something inherent in the short spark form of excitation, or a false effect, due to some peculiarity in the measuring instruments. The substitution of another voltmeter, which gave the same readings at all points which were tried, showed that the phenomenon was not due to any resonance with the natural period of the voltmeter pointer. Moreover, the high resistance of the voltmeter, amounting to about 60,000 ohms on the 600-volt scale, precluded the possibility of resonant oscillations through the voltmeter.

The measurement of the resistance in series with the arc at various currents and arc lengths furnished another test which seemed to be conclusive. In all the tests that were made, it was found that, within the limits of the errors of observation, the resistance of the circuit was the same, whether it was found from the ratio of the open circuit, E.M.F., E, to the current I, through the main circuit when the arc was short circuited, or from the formula,

$$E - V = RI, \tag{1}$$

where V is the voltmeter reading across the arc. The resistance in the main circuit corresponding to any current and arc length or voltage can be determined, therefore, to a close approximation from the curves of Fig. 7, if E is taken as 500 volts.

The characteristics from which the curves of Fig. 7 are taken were drawn as smooth curves in Fig. 5 and 6, all small deviations being taken as due to errors of observation. From an inspection of the points plotted in Fig. 3, however, it is apparent that there are systematic deviations in the neighborhood of 3 amperes, which would accentuate the maxima and minima in Fig. 7 slightly. A reexamination of all the original curves plotted showed that a similar effect ran through all the observations. In general these deviations are less than those shown at about 2.8 and 3.2 amperes in Fig. 3, and if they had been taken into account in plotting the final result would have been changed but little. At the longer arc lengths some small deviations gave rise to the suspicion that the minor maxima which are apparently just beyond the range of Fig. 7 came into this range at larger currents, but the effects were too small to make this certain.

The first clue to the cause of the phenomenon shown in Fig. 7 was obtained accidentally. Part of the variable resistance in the main circuit consisted of german silver coils, 61 in all, each about 61 cm. long, 1.5 cm. mean diameter and having about 2 turns per cm. of wire 1.13 mm. in diameter. The inductance of this resistance was about 330,000 cm., and it was found that the voltage reading for the same current and arc length varied when this resistance was replaced by the non-inductive rheostat. Thus, with an arc length of 0.2 mm. and arc current of 2.25 amperes, the voltmeter reading

rose from 59 volts to 64 volts when the inductive resistance was replaced by the rheostat, the intensity of the oscillations increasing with the voltage.

§ 6. *Variation of Voltage with the Capacity of the Arc.*

In seeking the reason for the variation of the arc voltage with the inductance of the main circuit, the relative times of charge and discharge of the condensers seemed to offer the most satisfactory basis for explanation. But the relative times of charge and discharge depend even more directly upon the capacity of the condensers, and experiments were made at once to determine if the capacity had any effect upon the arc voltage. It was found that increasing the capacity of the shunt circuit by adding more Leyden jars increased the intensity of oscillations, while the voltage sometimes increased and sometimes decreased. The d.c. ammeter reading would increase as the capacity was added, usually, and the resistance of the main circuit would have to be increased in order to bring it back to the same reading, but sometimes this effect would be reversed also.

These observations led to the suspicion that the curious variations in the arc voltage as the arc-length was increased might be due to the change in the capacity of the arc as the distance between the plates was varied. A rough computation, using 1 as the dielectric constant of the paper and gas between the plates, showed that the capacity of the arc varied between rather wide limits, from 0.0016 mfd. at 0.10 mm. arc-length to 0.00023 mfd. at 0.70 mm. Accordingly, a variable air condenser was connected directly across the arc, and the values of the arc voltage and oscillation current were read as the capacity in parallel with the arc was varied. The arc-length used was 0.25 mm., and the d.c. ammeter reading was maintained constant at 2.5 amperes by varying the resistance in the main circuit. The results are given in Table I. and graphically in Fig. 8.

In Fig. 8 the variable capacity in parallel with the arc is plotted as abscissa, with the d.c. voltage across the arc and the corresponding intensity of the oscillations indicated by a hot wire ammeter as ordinates. The capacities corresponding to arc-lengths of 0.05,

TABLE I.

Change of Arc Voltage and Oscillation Intensity with Arc Capacity.

Capacity in Parallel.	Volts, d. c.	Oscillation Current.	Resistance in Series.
0.000015 mfd.	64.5	12.4	176 ohms.
0.000075	68	12.8	174
0.000156	69.5	12.8	171
0.000195	73	12.5	171
0.000240	74	12.4	171
0.000285	77	11.1	166
0.000300	79	11.2	166
0.000320	80	11.2	166
0.000345	79	10.9	166
0.000375	77	11.0	168.5
0.000420	72	11.5	168.5
0.000560	70	12.6	174
0.00110	77	13.1	166
0.00176	83	13.2	158
0.00190	87	13.2	158
0.00248	96	13.4	155
0.00353	98	14.0	154

D.c. ammeter reading in main circuit maintained constant at 2.50 amperes by changing resistance. 500 volts open circuit.

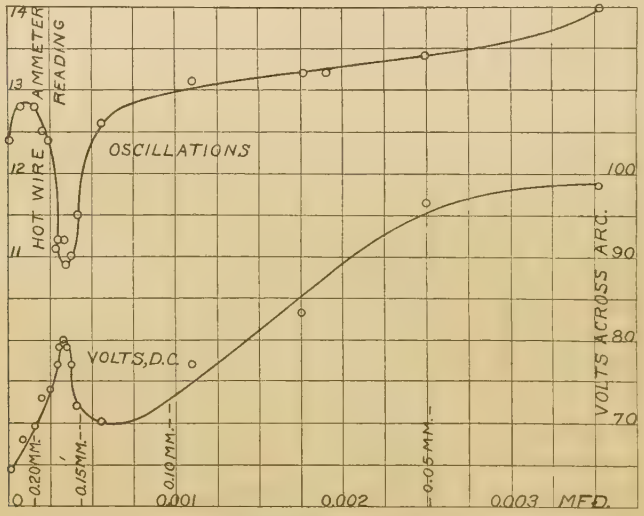

Fig. 8.

Effect of arc capacity on arc voltage and oscillations.

0.10, 0.15 and 0.20 mm. have been marked, and the left-hand boundary of the figure, with no capacity in parallel, corresponds of course to the arc-length of 0.25 mm. actually used. The resistances in series with the arc have not been plotted, but it is evident from Table I. that the resistance curve is approximately the reciprocal of the voltage curve. At the voltage maximum the resistance is a minimum, agreeing with the oscillations curve. At larger capacities, however, the resistance decreases with increasing voltage, whereas the oscillations increase. In all cases the value of the resistance observed agrees closely with the value obtained by subtracting the potential difference across the arc from 500 volts and dividing by 2.5 amperes, according to equation (1).

It is apparent that the voltage curve with varying arc capacity corresponds qualitatively at least with the voltage curve for varying arc length at 2.5 amperes shown in Fig. 7. The maximum in the voltage in Fig. 7 occurs at an arc-length less than 0.15 mm., while in Fig. 8 the maximum occurs at a capacity equivalent to an arc-length slightly more than 0.15 mm., as calculated. The discrepancy is due in part to the manner in which the plates forming the arc electrodes were arranged in the two experiments. In the earlier experiments which gave the data for the curves of Fig. 7 the plates could not be adjusted so as to completely overlap, on account of the projection of the screws which held the connecting wires. In the later experiments when the importance of the capacity of the arc had become apparent they were made to overlap more completely, thus increasing the capacity, and making an arc length of about 0.25 mm. in the later experiments equivalent to an arc length of about 0.20 mm. in Fig. 7. This would shift the maximum in Fig. 7 into the region of shorter arc lengths and tend to bring the two sets of curves into agreement.

The inductance of the wires which connected the variable air condensers in parallel with the arc would decrease the effect of their capacity and give a correction which would also shift the maximum in the right direction. This inductance was small, however, as the area enclosed by the connecting wires was not more than 3×25 sq. cm. The small correction due to a dielectric constant greater than one for the paper and gas between the plates

would make the equivalent capacity for each arc length slightly greater and would tend to neutralize in part the effects of the other two corrections.

The curve for the intensity of the oscillations in Fig. 8 shows that the oscillations are a minimum when the voltage is a maximum, *i. e.*, when the equivalent arc length is about 0.15 mm. The readings of the hot wire ammeter given in the table and shown graphically in the curve are in scale divisions of the instrument. The divisions were unequally spaced so as to give a simple multiplying factor when the ammeter was used unshunted, so that the readings may be taken as proportional to the oscillation current, even when used with a shunt, if, as in this experiment, the frequency is practically constant. In attempting to calibrate the instrument later by comparison with a less sensitive unshunted ammeter, the expanding wire was burned out. From the readings obtained however, and from observations of the intensity of the oscillations made under similar conditions later, the multiplying factor may be taken as about 0.4, so that a reading of 14 scale divisions indicated about 5.6 amperes.

As a further test of the effect of capacity on the arc voltage, a run was made with a constant d.c. ammeter reading of 2.0 amperes. The large upper electrode was replaced in this experiment with a small copper disc 3.85 cm. in diameter and an arc length of 0.15 mm. was used, giving a capacity of only 0.000069 mfd. as compared with 0.00108 mfd. for the larger plate with the same arc-length. On varying the capacity of the air condensers in parallel with the arc, it was found that the maximum in the voltage had shifted to a smaller capacity, corresponding to a longer arc length of slightly more than 0.2 mm. This is in agreement with the shift of the maxima to longer arc lengths with smaller currents shown in Fig. 7.

In this experiment it was possible to reduce the capacity of the arc to smaller values than had been obtained before, and it was found that the voltage decreased from a maximum of 100 volts at a capacity equivalent to about 0.2 mm. to a minimum of 80 volts at 0.4 mm. equivalent, and then increased to another maximum of 85 volts at a capacity 0.9 mm. equivalent, falling off to 75 volts

when there was no capacity in parallel with the small electrode. The second maximum observed evidently lies just beyond the range of Fig. 7. A minimum of 4.7 amperes was observed in the oscillations when the voltage was at the first maximum corresponding to 0.2 mm., although the minimum in the curve for the oscillations was not so sharp as that shown in Fig. 8. There was no minimum in the oscillations corresponding to the smaller second maximum in the voltage at 0.9 mm. equivalent, but instead a maximum of 5.3 amperes, the reason for which is unknown.

At 1.5 amperes the maximum in the voltage shifted over to the region of still smaller capacities, corresponding to a longer arc length between 0.20 and 0.25 mm., and the maximum increased to 113 volts. Both the shift and the increase in the maximum are in agreement with Fig. 7.

The experiments with capacity in parallel with the arc show therefore that the maxima observed in the voltage in Fig. 7 are due chiefly to the changing capacity of the arc itself, as the arc length is varied, and that corresponding to these maxima in the voltage there are minima in the oscillations produced by the arc. This is in agreement with the hypothesis advanced to account for the maxima in the voltage-arc length curves. When the time required for the main current to charge the condensers is long compared with the time of discharge, the effective value of the oscillations should be small and the average value of the d.c. voltmeter reading large. The reason for the minima in the oscillations and the corresponding maxima in the voltage together with the importance of these minima in the production of impact excitation effects will be considered more fully in the later sections which deal especially with the intensity of the oscillations. A brief summary of the explanation, however, is given in section 7.

From the connection which has been established between the maxima in the voltages and the minima in the oscillations it is possible to predict what would be the effect upon the curves in Fig. 7 if the capacity or inductance of the condenser circuit were varied. It will be shown in the sections dealing with the intensity of the oscillations that the d.c. ammeter reading at which the minima in the oscillations occur increases with increasing capacity

and with decreasing inductance in the condenser circuit. This is the same effect as shown in Fig. 7, where the corresponding maxima in the voltage moves into the region of smaller arc-length and larger arc capacity as the d.c. ammeter reading is increased. We may consider, then, that the capacities of the condensers and of the arc are in parallel, and that the effective capacity of the condensers is decreased by the inductance in series with them. If the curves shown in Fig. 7 were determined with a larger capacity or a smaller inductance in the shunt circuit, therefore, the capacity of the arc would have to be reduced in order to have the same capacity for the whole system, and the maxima would move into the region of longer arc-lengths.

The reason for the increase of voltage observed when the inductance of the main circuit was decreased becomes clear when we consider the effect upon the relative times of charge and discharge. With a smaller inductance in the main circuit the generator current falls off more during the condenser charge, and the time of charge is made longer. But this is the same effect as that produced by an increase of capacity. Decreasing the inductance of the main circuit should have the same effect as increasing the capacity or decreasing the arc length, therefore, and from Fig. 7 it is apparent that at 0.2 mm. and about 2.25 amperes a decrease of arc-length should raise the voltage. This is in agreement with the effect observed. A change of inductance in the main circuit should produce an increase of voltage if made on one side of the maximum and a decrease if made on the other, and this effect has also been observed in the author's experiments, always in the direction which agrees with the theory.

§ 7. *Explanation of Maxima and Minima.*

At small capacities the condensers will charge up quickly, the number of discharges per second will be great, and the sparking potential will be low, since the gas will remain ionized to a large extent. The intensity of the oscillations, as measured by a hot wire ammeter, will be large, and the average d.c. voltmeter reading low, partly on account of the low sparking potential, and partly because the charging time, when the voltmeter reading is obtained,

is relatively small compared with the discharging time, when the voltmeter indication should be relatively small.

At large capacities, on the other hand, the quantity of energy liberated at each discharge is large, the trains of oscillations will last a long time, and their effective value will be large even though the number of discharges per second is relatively small. A large reading of the hot wire ammeter should therefore be obtained. But the resistance of the arc will be small while the large current is flowing through it and the time of each discharge is large compared with the time of charge. In this case also, then, the average voltmeter reading will be small.

Between these limiting cases a low value of the oscillations may result from a low value of the group frequency, due to high sparking potential, and a low value of effective oscillation current, due to small capacity. The time of charge will be relatively great and the time of discharge relatively small, giving a large value of the average voltmeter reading. The minimum in the oscillations should correspond with a maximum in the voltage, therefore, and this effect is observed in Fig. 8.

The reason for the shift of the maxima into the region of larger arc capacities as the current is increased is clear from this explanation. If the direct current from the generator is increased, the time required to charge the condensers will be decreased. This is the same effect as would be produced by a decrease of capacity. To neutralize this effect the actual capacity would have to be increased, thus restoring things to the same condition. In other words as the charging current is increased the maxima should move over into the region of larger capacities or shorter arc-lengths, and this is the effect found experimentally.

The combination of all the independent and dependent variables, capacity, sparking potential, times of charge and discharge and group frequency, may give rise to several maxima and minima when the capacity is varied and the current is maintained constant. A complete discussion of these can only be obtained with the aid of the equations of charge and discharge after the form of the sparking potential as a function of the effective arc current, the capacity and the other constants of the circuit are known.

The rapidity with which the oscillations are damped out will evidently be a function of the maximum amplitude with which they start, depending upon the primary capacity and inductance, and of the sparking potential and group frequency. The minimum value of the oscillations corresponds both to relatively small initial amplitude and high sparking potential and therefore to strong quenching. The points of the maximum voltage readings in Fig. 7, which correspond to minima and strong quenching in the oscillations, represent conditions especially favorable for the production of impact excitation effects.

§ 8. *Variation of Sparking Potential with Arc Current.*

In order to test the assumption that the sparking potential required to break down the insulation increased as the arc current was diminished, a Braun cathode ray oscillograph was connected across the arc, as shown in Fig. 9, so that the maximum deflection of the ray by the electromagnetic field measured the maximum instantaneous or sparking potential across the arc.

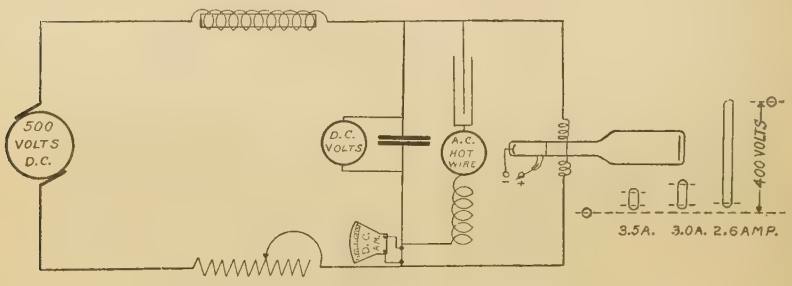

Fig. 9.

Determination of sparking potential by Braun oscillograph.

The oscillograph was calibrated by noting the deflection produced by different applied voltages before the arc was started, and it was found that the potential difference measured by the voltmeter across the arc was directly proportional to the deflection of the fluorescent spot on the cross-sectioned screen. It was found that a good deflection of 21 scale divisions could be produced by one half an ampere, and a resistance of 450 ohms was placed in

series with the deflecting field before it was connected across the arc. On account of the appreciable current taken by the deflecting field, the five ampere shunt which converted the millivoltmeter into a d.c. ammeter was connected so that only the arc current passed through it.

At the right of Fig. 9 is shown the appearance of the fluorescent spot under the different conditions of the experiment. With no potential difference across the arc the fluorescent spot was a small circle, as shown at the left. When the current of 3.5 amperes d.c. was sent through the arc and the oscillations started, the circle spread out into a band of light with bright spots at the ends and a less bright region between. The lowest position of the spot of light corresponded to 29 volts across the arc, and highest position to a maximum potential of 68 volts. The voltmeter reading was between these limits, 52.5 volts, and the intensity of the oscillations was about 5.4 amperes.

When the d.c. ammeter reading was reduced to 3.0 amperes, the bright discs spread still farther apart, the lower one rising slightly to 31 volts, and upper limit increasing to a maximum potential of 104 volts. The voltmeter reading was 68 volts, and the intensity of the oscillations 5.6 amperes.

When the current was reduced still further to 2.6 amperes d.c., bringing it into the region of the oscillation minimum and voltage maximum, the lower disk was raised to a point corresponding to 34 volts, but the well defined upper disk disappeared. A bright region extended up to something more than 100 volts, and beyond this frequent long streamers or tails reached up to 400 volts or more. The voltmeter reading was about 76 volts, occasionally running up to 80 volts, and the intensity of the oscillations fell off to 5.0 amperes. Finally the arc went out, and the impressed potential difference across the plates was found to be 400 volts.

No oscillations could be obtained at smaller currents, nor could the arc be kept in operation. Apparently the shunt of 450 ohms across the arc prevented the high sparking potentials which were necessary at smaller currents from being attained. An arc-length of 0.15 mm. was used in all cases, and the capacity and inductance of the oscillatory circuit were 0.0169 mfd. and 14,400 cm. respect-

ively. The experiments were repeated with a capacity of 0.0132 mfd. and gave very nearly the same results for the large currents, while at 2.7 amperes the maximum voltage as shown by the streamers on the fluorescent screen was about 300 volts.

The experiment shows therefore, that the maximum or sparking potential increases as the arc current is decreased and is accordingly in agreement with the theory advanced. Apparently at the point where the oscillations are a minimum there is a cumulative effect which increases the sparking potential more for a given decrease of direct arc current than at other points. This is due undoubtedly to the effect of the oscillations themselves upon the sparking potential. The lowest potential across the arc also increases slowly as the direct current is increased, and apparently agrees fairly well with the curve marked "without oscillations" in Fig. 4, the voltages being slightly lower in this case because the oscillations lower the resistance of the arc.

All the phenomena observed with the cathode ray oscillograph pointed to a form of voltage oscillation curves very similar to those observed by Barkhausen[1] using the Poulsen arc. He employed an electric field to give a deflection proportional to the arc voltage, and a magnetic field to give a deflection at right angles proportional to the condenser current. His curves show large increases of the sparking potential as the arc current is decreased, and smaller increases of the lower limit of potential, in agreement with the author's experiments. Unfortunately, no plates for making an electric deflecting field were available inside the Braun tube used by the author. It is evident, however, that the phenomena observed in one dimension in the author's experiments on the Lepel arc are very closely related to the boot shaped diagrams drawn out in two dimensions in Barkhausen's curves.

§ 9. *Variation of Arc Current with Arc Length.*

If horizontal sections are taken through the family of curves shown in Figs. 5 and 6 the variation of the current through the arc, as read by the d.c. ammeter, is found to have maximum and mini-

[1]H. Barkhausen, "Die Erzeugung dauernder Schwingungen durch den Lichtbogen," Jahrbuch der drahtlosen Telegraphie und Telephonie,' Vol. I., p. 243, 1907.

mum values as the arc length is increased while the voltage across the arc is maintained constant. This is chiefly an effect due to the capacity of the arc, and is explained in the same way as the variation in voltage at constant current was explained, the two phenomena being complementary. The curves obtained in this way are shown in Fig. 10.

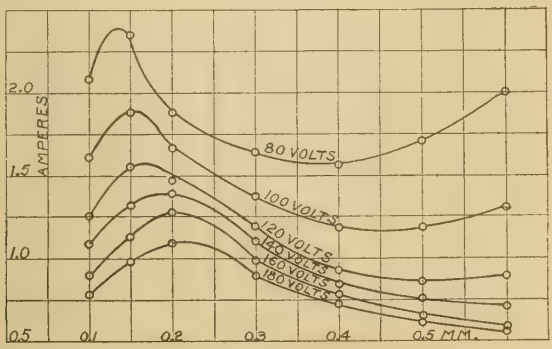

Fig. 10.

Variation of arc current with arc length at constant voltage.

It is apparent from a comparison of Figs. 7 and 10 that the maxima for the volts at constant amperes correspond to the maxima for the amperes at constant volts, all the quantities being measured with d.c. instrument. In both sets of curves the maxima move into the regions of longer arc-lengths as the voltage is increased or the current reduced.

An alternative interpretation may be given to the curves in Fig. 7 and 10 if we assume that the voltage across the arc depends not merely upon the direct current through the arc, but rather upon the effective arc current, or the square root of the sum of the squares of the direct current and the oscillatory current. According to this interpretation, when the oscillations are at a minimum, corresponding to a critical value of the capacity of the arc, the effective current through the arc would be small, and the voltage across the arc would rise to a maximum even though the d.c. ammeter reading is kept constant. The maxima in Fig. 10 correspond to the minima oscillations, and it is apparent, from the

point of view of effective current values, why the direct current must be increased in order to keep the voltage constant.

According to this interpretation, if the effective arc current, as measured by a hot wire ammeter, were maintained constant, the voltage across the arc should remain constant when the capacity across the arc is varied, and should change but slightly when the arc length is varied.

§ 10. *Analogous Effects Observed by Duddell and Others.*

Investigators who have used the ordinary spark gap as a means of producing high frequency oscillations have long been familiar with a phenomenon similar to that shown by the curves in Fig. 10. For a given constant applied voltage at the high tension transformer or induction coil, there is a definite spark length which will give a maximum current in all the circuits. It is possible that this effect is due in part to the influence of the capacity of the spark, but it is probable that the variation of the sparking voltage and spark resistance with the spark length plays a much more important than in the case of the short arc between large electrodes.

A more marked effect has been observed by Duddell[1] in the secondary current of an induction coil when a mercury interrupter was used to make and break the primary current. With a small spark gap between two aluminium points about 1.2 mm. apart in the secondary, the current measured by a hot wire ammeter rose to a maximum about double the value observed when the secondary spark gap was short circuited. The maximum in Duddell's curve with currents as ordinates and spark lengths as abscissas is sharper than that shown in any of the curves in Fig. 10, but the general characteristics are the same. Duddell's curve has also the same features of a maximum followed by a minimum and then an increase of current as far as the observations are continued shown by the upper curves in Fig. 10. The influence of a change in the shape of the electrodes in Duddell's experiment and the variations in the oscillations when the capacity of the secondary circuit was changed by touching an insulated metal plate to various points of it indicate that the effect was due in part at least to the capacity of the spark

[1]W. Duddell, The Electrician, Vol. 61, p. 796, 1908. See also discussion, p. 134.

gap. It is probable, however, that the resistance of the spark gap in Duddell's experiment plays a relatively more important part than in the phenomena of the Lepel arc.

Summary of Results.

For electrodes of the short arc oscillation generator copper was found to be most satisfactory material, and with plates not more than one quarter inch thick water cooling was found to be unnecessary. The kind of paper used for separating the plates is immaterial, but much difficulty attends the use of mica. Charring the paper and breaking a short circuit across the arc in oil is the most effective method of starting the arc when only a low voltage is available. Connecting arcs in parallel, each with its own choke coil and inductance, gives double power for the same applied voltage, and has other advantages over series connection.

The Lepel arc has a falling characteristic much steeper than those of the Duddell or Poulsen arcs and operates on smaller currents. The separation of the two characteristics obtained with and without oscillations is much greater for the Lepel arc than for the others. The characteristics for increasing arc-lengths lie one above the other at large currents, but at small currents the order is reversed. This is found to be due to marked maxima in the voltage across the arc which occur at smaller arc lengths as the current is increased.

The maxima in the voltage are due indirectly to the variation in the arc capacity, which is quite large on account of the small distance between the plates which form the electrodes. The maxima in the voltage are caused by corresponding minima in the oscillations produced by the arc. The capacity of the arc and of the condensers are in parallel, but the inductance in series with the condensers partly neutralizes the effect of their capacity and makes the capacity of the arc more important. Decreasing the inductance of the main circuit has the same effect on the voltage as increasing the capacity of the arc.

The minima in the oscillations and corresponding maxima in the voltage across the arc occur because for certain values of sparking, potential, arc current, capacity and inductance, the time of discharge is a minimum compared with the time of charge. The minima in

the oscillations furnish conditions especially favorable for strong quenching of the oscillation and for impact excitation effects.

Experiments with Braun cathode ray tube show that the maximum instantaneous or sparking potential for the arc increases as the arc current is decreased. The sparking potential is especially high in the region of the minima in the oscillations.

The direct current through the arc required to maintain the voltage constant as the arc-length is varied is found to have a maximum where the oscillations are a minimum. This is chiefly a capacity effect also, but is similar to certain phenomena in spark gap excitation in which the variation in the resistance of the arc plays a more important part.

In the succeeding sections on the intensity and frequency of the oscillations, it will be shown that the minima in the oscillations occur at larger values of the direct current through the arc as the capacity in the shunt circuit is increased or the inductance diminished, and that the intensity of the oscillations themselves increase. It will also be shown that the frequency of the oscillations increases with the arc current and decreases with the arc length, and that the formula derived by the author for the frequency of the singing arc holds true also for the frequency of the oscillations produced by the new short arcs between metal electrodes.

Physical Laboratory,
Cornell University,
June 25, 1910.

EXPERIMENTS IN IMPACT EXCITATION.

By George W. Nasmyth.

II. The Intensity of the Lepel Arc Oscillations as a Function of the Arc Current, Capacity and Inductance.

IN the preceding sections on "The Characteristics of the Short Arc Metal Electrode Generators" the apparatus used and the results of the experiments on the relation between the voltage and current of the Lepel arc, consisting of two copper plates separated by a few sheets of paper, were described. It was found that the Lepel generator resembles the arc more closely than the spark gap, and has, in common with the Duddell and Poulsen arcs, a falling voltage-ampere characteristic. Certain curious maxima in the d.c. voltage across the arc were observed as the arc length was varied at constant current, and the cause of these was finally traced to the influence of the capacity of the arc itself. It was found that these maxima in the voltage are accompanied by corresponding minima in the intensity of the oscillations. In the following sections the effect of the variation of the arc current and the capacity of the condenser circuit upon the intensity of the oscillations will be investigated, and later the question of the frequency will be taken up. For the present the connections remain as shown in Fig. 1, the Lepel arc being connected to a 500-volt d.c. generator through a choke coil and variable resistance, with an oscillation circuit containing capacity, inductance and a thermoammeter, in shunt.

§ 11. *The Intensity of the Oscillations.*

Two striking phenomena meet the observer who sets out to investigate the intensity of the oscillations in the condenser circuit. First, the alternating current, as measured by an unshunted hot wire ammeter in the condenser circuit, is two or three times greater than the direct current measured by an ammeter in the main circuit.

Second, if the direct current in the main circuit is varied, the intensity of the oscillations passes through a well-defined minimum. The value of the direct current at which this minimum in the oscillations occurs depends on the constants of the circuit, increasing when the capacity is increased, and decreasing when the inductance is increased. The first characteristic indicates that these oscillations belong to the third class of "Wien" or "impact" oscillations; the second characteristic has interesting consequences in the theory of the relation of the intensity of the oscillations to the other factors of the circuit.

Inasmuch as the frequency of the oscillations varies with the current in the main circuit, the difficulties of calibration at varying frequencies made it impossible to use any kind of an ammeter with a shunt. Accordingly a special ammeter was constructed without the use of iron, in which the entire current passed through a small copper wire, 0.22 mm. in diameter, and about 9.5 cm. long. This ammeter was connected in the oscillation circuit, as shown by the dotted lines in Fig. 1 of the preceding article (see p. 76), and calibrated before and after each set of observations by comparison with the direct current ammeter in the main circuit.

The results of the experiments on the variation in the intensity of the oscillations with the arc current are given in Table II. and graphically in Fig. 11. The readings of the d.c. ammeter in the main circuit, called for brevity arc amperes, were varied by changing the resistance in the main circuit and the corresponding oscillation amperes as indicated by the hot wire ammeter in the condenser circuit were noted. The corresponding readings of the d.c. voltmeter connected to the two electrodes of the Lepel arc are also given.

Each curve in Fig. 11 corresponds to a definite capacity in the oscillation circuit, one Leyden jar representing a capacity of 0.00188 microfarad. The same inductance, 14,400 cm., was used for the nine curves, so that the frequency range was from 324,000 with 9 jars capacity, to nearly a million with one jar.

In each of the curves in Fig. 11 the oscillations pass through a minimum, well-defined in the curves for the larger capacities, less so at small capacities. These minima move over into the region of larger arc currents as the capacity is increased, following ap-

Table II.

Variation of Oscillation Intensity with Arc Current and Primary Capacity. Arc Length 0.15 mm. $L_1 = 14{,}400$ cm. Copper electrodes six inches diam.

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
Capacity, 1 Jar=0.00188 mfd.		
0.50	1.00	——
0.70	1.20	176
1.00	1.40	108
1.15	1.70	94
1.25	1.70	82
1.34	1.86	84
1.60	2.00	72*
1.70	1.81	64
1.75	1.65	63
1.85	1.65	62
1.90	1.63	62
2.00	1.65	61.5
2.20	1.86	60
2.40	2.03	57
2.50	2.06	57
Capacity, 2 Jars=0.00376 mfd.		
0.76	2.40	——
1.03	2.20	——
1.50	2.32	76
1.55 min.	2.28	71
1.60	2.32	70
1.70	2.43	69
1.80	2.49	68
1.90	2.64	66
2.00	2.76	64
2.50	2.34	63*
2.65	2.30	59*
Capacity, 3 Jars=0.00564 mfd.		
1.30	3.00	——
1.41 min.	2.80	91
1.45	2.90	88
2.20	3.03	64
2.30	3.05	63
2.40	3.10	62
2.50	3.21	62
2.60	3.28	61

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
Capacity, 4 Jars=0.00752 mfd.		
1.45	3.85	96
1.60	3.25	82
1.70	3.17	79
1.75 min.	3.12	76.5
1.80	3.14	75.5
2.00	3.25	70
2.05	3.30	69
2.25	3.41	67
2.50	3.48	62
2.71	3.52	56
2.75	3.53	56
2.90	3.54	54.7
3.01	3.69	53
3.20	3.90	52
3.30	3.86	52
3.36	3.90	52
3.42	3.95	52
3.50	3.80	51.5
3.63	3.59	48
3.72	3.69	49
3.80	3.55	47
4.00	3.70	46
Capacity, 5 Jars=0.00940 mfd.		
1.08	5.56	——
1.30	5.15	146
1.48	5.00	126
1.60	4.92	116
1.70	4.62	104
1.80	4.27	88
2.00	3.82	73
2.15	3.80	70
2.25	3.57	66*
2.50	3.77	65
2.75	3.86	61
3.00	3.90	53*
3.26	4.01	50
3.50	4.12	50

Arc Amperes D.C.	Oscillation Amperes.	Arc Volts D.C.
Capacity, 6 Jars=0.01128 mfd.		
1.45	6.23	——
1.52	6.05	138
1.63	6.05	128
2.00	5.70	97
2.23	5.00	82
2.47 min.	4.04	68
2.54	4.09	——
2.67	4.25	64
2.75	4.35	63
2.94	4.62	59
3.00	4.76	58
3.25	4.76	56
3.50	4.83	53
3.75	4.72	49
4.00	4.83	47
Capacity, 7 Jars=0.01316 mfd.		
2.35	6.00	
2.50	5.83	
2.55	5.60	
2.75	5.24	
2.83	5.20	
2.97 min.	4.92	
3.02	4.98	
3.20	5.14	
Capacity, 8 Jars=0.01504 mfd.		
2.35	6.28	
2.75	6.00	
3.02	5.60	
3.05	5.50	
3.19	5.28	
3.29 min.	5.10	
3.45	5.35	
3.50	5.24	
Capacity, 9 Jars=0.0169 mfd.		
2.82	6.1	72
3.18	5.9	—
3.43	5.6	—
3.51 min.	5.2	50
3.80	5.6	50

Points starred * are not plotted, to avoid confusion in the figure.

parently a straight line law. The descent to the minima is much steeper from the region of smaller arc currents, and in some of the curves a maximum followed by another minimum can be seen in the region of larger arc currents.

It was shown in § 7 that there are maxima in the voltage across the arc corresponding to minima in the oscillations as the capacity

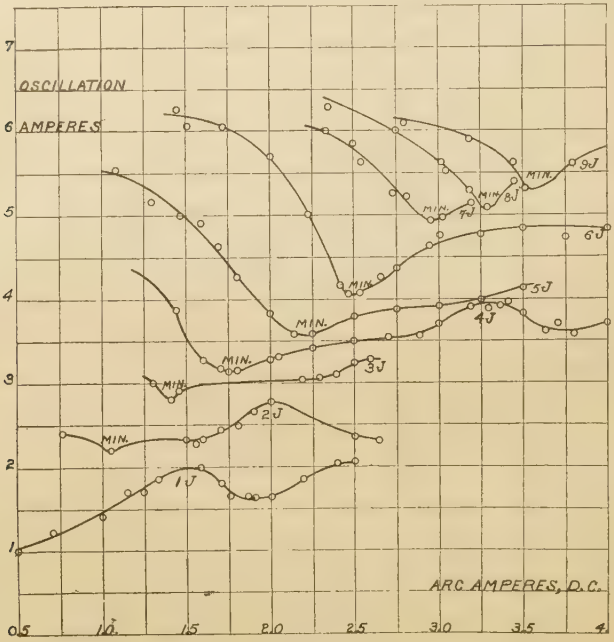

Fig. 11.

Variation in intensity of oscillations with arc current at constant capacities.

of the arc is varied at constant arc current. Moreover in § 5 it was shown that as the capacity of the arc was increased the maxima in the voltage moved into the region of larger arc currents.

This is evidently another phase of the phenomena shown by the shifting of the minima to larger arc currents as the capacity is increased, in Fig. 11, and furnishes additional evidence that the capacity of the arc itself must be considered as connected in parallel

with the capacity of the condenser-circuit as far as the effect upon the intensity of the oscillations is concerned.

The effect of the primary capacity upon the arc voltage is shown in Fig. 12, from the data of Table II. Besides the increase of voltage with added capacity there is also a change of the form and slope of the curves, the increase of voltage for small currents being greater than that for large currents. There is some indication of the maxima in the voltage corresponding to the minima in the

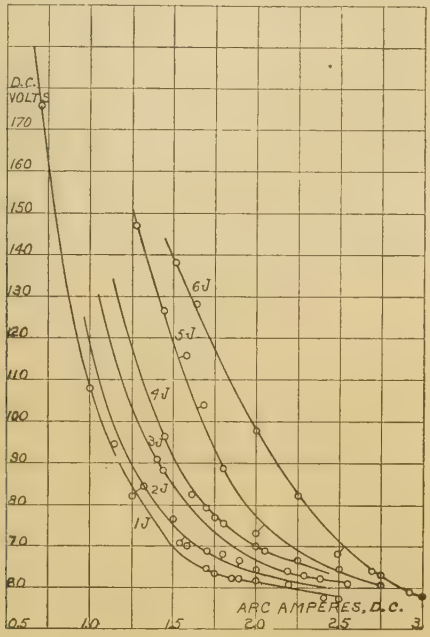

Fig. 12.
Increase of arc voltage with primary capacity.

oscillations of Fig. 11, but not enough points are available to bring out the humps of the curves clearly.

The influence of the minima in the oscillation intensity upon the frequency of the oscillations will be discussed in §§ 17–19. The bearing of the effect of capacity in increasing the oscillation intensity and changing the slope of the voltage-ampere characteristics upon the question of the frequency will appear in § 20.

In Table III. arc currents at which the minima in the oscillations in Fig. 11 occur, together with the intensity of the oscillations corresponding, are tabulated. Similar data are given for larger inductances with the addition of the d.c. voltmeter reading across the arc. In the data for 46,400 cm., the readings when a non-inductive resistance of one ohm was added to the condenser circuit are also given. The effect of resistance seems to be the same as that of capacity in increasing the arc voltage and the arc current at which the oscillation minima are found, but the change is too small to be conclusive.

TABLE III.

Shift of Oscillation Minima with Capacity and Inductance. Arc Length 0.15 mm

Primary Capacity Leyden Jars. Capacity, Microfarads.	3 0.00564	4 0.00752	5 0.00940	6 0.01128	7 0.01316	8 0.01504	9 0.0169
Primary Inductance $L_1 = 14{,}400$ cm.							
Arc Current for Minimum Oscillations	1.33	1.84	2.12	2.34	3.02		
Minimum Oscillation Amperes	3.1	3.6	4.06	4.10	4.98		
Primary Inductance $L_1 = 46{,}400$ cm.							
Arc Current for Minimum Oscillations	1.22	1.60	1.85	2.20	2.60	2.95	
Same, one ohm added	1.32	1.68	2.00				
Minimum Oscillation Amperes	3.08	3.23	3.42	3.51	3.90	4.08	
Same, one ohm added	3.04	3.27	3.51				
Arc Volts at Minimum Oscillations	64	56	46	41	41	—	
Same, one ohm added	76	64	64.2				
Primary Inductance $L_1 = 176{,}000$ cm.							
Arc Current for Minimum Oscillations		1.45	1.68	1.98	2.25	2.65	2.90
Minimum Oscillation Amperes		—	1.49	1.77	1.99	2.46	2.95
Arc Volts at Minimum Oscillations		—	45	44	42.5	42	44
Primary Inductance $L_1 = 327{,}500$ cm. Lower Minima.							
Arc Current for Minimum Oscillations		1.02	1.22	1.40	1.68	1.83	1.95
Minimum Oscillation Amperes		1.57	1.82	1.90	2.03	2.19	2.35
Arc Volts at Minimum Oscillations		76	66	58	53.5	49.5	47.5

In Fig. 13 the arc currents at which the minima in the oscillations occur are plotted as ordinates with the corresponding capacities as abscissas. The resulting curves are seen to be straight lines. As the inductance in the oscillation circuit is increased, the slope of these lines is seen to decrease, at first rapidly, then more slowly. The arc current for minimum oscillations is apparently some inverse function of the inductance, and directly proportional to the capacity.

At small inductances traces of a second minimum were observed at smaller currents, and at large inductances these secondary minima became more prominent than the others. The lowest curve shows

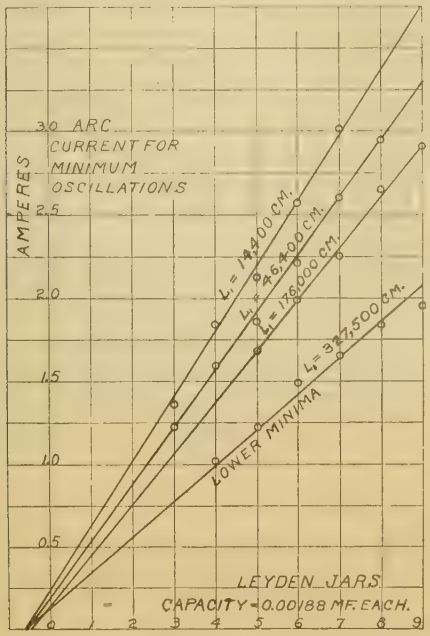

Fig. 13.

Shift of oscillation minima to larger arc currents with increasing capacity and decreasing inductance.

that this secondary minimum follows the same straight line law as the primary minima.

§ 12. *Influence of the Capacity of the Arc.*

The straight lines of Fig. 13 all give an x intercept corresponding to the capacity of the arc itself, considered as a condenser. The capacity of the arc has been computed from the dimensions and distance apart of the plates to be about two thirds of one of the Leyden jars. The mean value of the x intercepts in Fig. 13 by least squares was found to be 0.54 Leyden jars, equivalent to 0.01

microfarad. In the experiments the plates never overlapped completely, on account of the projection of the screws and connecting wires, and it was observed that when the overlapping was less complete, as in the experiments at 176,000 cm. of inductance, the x intercept corresponding to the capacity of the arc was smaller.

The capacity of the arc is added to the capacity of the condenser circuit as far as the intensity of the oscillations is concerned. But for the frequency of the oscillations the author's experiments, to be described later, show that the arc acts merely as a resistance and that its capacity effect does not enter directly. This is only to be expected, as the arc could hardly act as a condenser when its insulation is broken down and a current is flowing through it. An important corollary of these two observations is that the Lepel arc may be available as a source of very high frequency oscillations whose intensity shall not be too greatly decreased. In other words, with an arc whose capacity is large, it may be possible to produce electric waves shorter than one meter, having a mean intensity much greater than that of the short waves produced by a Hertz oscillator.

§ 13. *Theoretical Considerations.*

We can obtain some insight into the reason for the minima observed in the intensity of the oscillations and shown in Fig. 11 with the aid of a formula first derived by M. La Rosa[1] for the maximum value of the oscillatory current. If we write down the energy relations for the condenser circuit for the instant of sparking and for the instant when the condenser current is a maximum, we have

$$\frac{Li_{\max.}^2}{2} = \frac{LI^2}{2} + \frac{Ce^2}{2}$$

where i is the condenser and I the generator current and e the sparking potential of the gap.

This equation states that the energy stored in the self induction ($\frac{1}{2}Li_{\max.}^2$) at the instant when the current in the condenser is a maximum is equal to the energy contained in the self induction ($\frac{1}{2}Li^2$) at the instant of sparking increased by the energy given to

[1] La Rosa, Nuovo Cimento, Vol. 7, 5, 1904.

the condenser ($\frac{1}{2}Ce^2$) during the charging time T_1. Solved for the maximum condenser current, the equation becomes

$$i_{max.} = \sqrt{(I^2 + Ce^2/L)}. \qquad (5)$$

The four assumptions under which the formula holds have been summarized by Vollmer as follows:

1. At the instant of sparking the condenser current $i = I$, the generator current.

2. The damping loss due to the resistance of the condenser circuit is negligible.

3. The energy produced in the condenser circuit on account of the falling characteristic of the arc during the discharge time T_2 is negligible. (In the Duddell oscillations this is the main source of energy.)

4. The energy $Ce_b^2/2$, stored in the condenser at the instant that the condenser current is a maximum is negligible. (e_b is the potential difference across the arc during the maximum discharge.)

In order that the formula shall hold for the short spark generators it is necessary to assume also:

5. The energy stored in the capacity of the spark gap itself is negligible.

Theory shows that the assumption 1 is permissible since for a sufficiently large choke coil in the main circuit the changes in the generator current are very small.

The errors introduced by assumptions 2 and 3 tend to neutralize each other, since the amounts of energy neglected in the two cases have opposite signs. Only the difference in these amounts of enegy will have an influence and this difference will usually be small.

Assumption 4 is unnecessary, as Vollmer has shown, if we use, instead of e^2, the difference of the squares of the sparking potential e and the potential difference across the arc during discharge, e_b.

The author's experiments have shown that the capacity of the arc cannot be neglected in the short spark generators, but as a first approximation we may treat the equation as it stands.

The experiments with the Braun tube described in § 8 have shown that the sparking potential decreases as the generator current I is

increased. Barkhausen[1] has found the same effect in the Poulsen oscillations. The sparking potential also increases with the arc length, as might be expected. The data available are not sufficiently accurate or extended to determine the law of the variation of sparking potential with the arc current, but if we assume the simplest relation, that the sparking potential is inversely proportional to the generator current, we have

$$e^2 = k/I^2,$$

the proportionality factor k being a function of the arc length. For small variations in the arc length is it probable that k may be considered a constant, as the variation in k will be small in comparison with the total sparking potential even at very small lengths. Using this value of e, the approximate formula (5) becomes

$$i_{\max.} = \sqrt{(I^2 + Ck/LI^2)}. \tag{6}$$

At large values of the generator current I, the first term under the radical will be of importance, and the second term small. For small values of the generator current, the second term will be important, and the first term small. Under the limiting conditions of very large or very small currents I, therefore, the condenser current maximum, $i_{\max.}$ will be large, but at intermediate values the condenser current will have a minumim in its initial maximum value and in its corresponding effective value. This is in agreement with the experiments of the author.

We can find the value of the current for which the oscillations will be a minimum by equating the differential $di_{\max.}/dI$ to zero. The condition for a minimum is

$$I^4 = k\frac{C}{L}, \tag{7}$$

so that the critical current for minimum oscillations should be a function of the capacity and inductance at constant arc length. This also is in agreement so far with the author's experimental results. The experiments show that the critical current is directly proportional to C (Fig. 13) instead of to the fourth root, however.

[1]H. Barkhausen, Jahrbuch der Drahtlosen Telegraphie und Telephonie, Vol. 1, p. 251.

The difference may be due in part to the influence of capacity on the relations of the maximum and effective values of the condenser current, and in part to the influence of the frequency, which is a function of the capacity on the sparking potential. The indirect effect of an increase in the capacity in lowering the sparking potential (through the influence of the duration and intensity of the oscillatory current) would also modify the relation between the capacity and arc current for minimum oscillations.

The capacity of the arc is in parallel with the capacity of the condensers and the divided circuit may give rise to the two minima, observed in the oscillations. The effective capacity of the condenser branch of the divided circuit is reduced by the inductance in series, however, and with a large inductance in the condenser circuit the capacity of the arc becomes more important. This is probably the reason why the second minima become more important than the first at the large inductance of 327,500 cm. as shown in Fig. 13.

Impact Excitation.—The bearing of the minima in the value of the effective and initial maximum condenser current upon the problem of impact excitation is apparent. It is evident that the oscillations will be most quickly damped out when their initial value is least. The average intensity of the oscillations will be least also in this case. To secure the most favorable conditions for impact excitation, therefore, the arc current or arc length should be adjusted to give a minimum in the intensity of the oscillations. If the arc current is varied by means of a rheostat in the main circuit at constant arc length, capacity and inductance, the desired minimum can easily be found with the aid of a hot wire ammeter.

Summary.

The intensity of the oscillations in the condenser circuit of the Lepel arc is in general two or three times greater than the direct current supplied to the arc by the generator.

The intensity of the oscillations increases with the capacity very rapidly at small arc currents, more slowly at larger currents. Inductance and resistance decrease the intensity of the oscillations.

As the arc current is increased at constant capacity the oscillations

fall rapidly to a minimum and then increase slowly, passing through a maximum to a secondary minimum.

As the capacity is increased or the inductance decreased, the minima and maxima in the oscillations shift into the region of larger arc currents. A linear relation exists between the arc current for minimum oscillations and the capacity. The range through which oscillations can be obtained shifts with the minima.

The explanation for the minima in the oscillations is found in the effects of the energy stored at the instant the gap breaks down. The energy stored in the inductance is large for large arc currents, while the energy stored in the capacity is large, on account of the increased sparking potential, at small arc currents, giving a minimum at intermediate arc currents.

The condition of minimum oscillations is most favorable for strong quenching and impact excitation.

The d.c. voltage across the arc increases with the capacity of the condenser circuit. The increase is most rapid at small arc currents, so that the form of the voltage-current characteristics is changed by the capacity in parallel.

The capacity of the arc is added, as if it formed a parallel circuit to the capacity of the condenser circuit, for the intensity of the oscillations, but not for the frequency.

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EXPERIMENTS IN IMPACT EXCITATION.

BY GEORGE W. NASMYTH.

III. THE FREQUENCY OF THE LEPEL OSCILLATIONS.

§ 14. *Importance of the Variation in Frequency.*

THE frequency of the oscillations produced by the short-arc generators is of considerable practical importance on account of the effect of variations in the frequency upon the sharpness of tuning which can be obtained. The frequency of the oscillations produced by the singing arc is not determined by the inductance and capacity of the shunt circuit alone. With the Duddell arc the variations in the pitch caused by a change in the arc current or arc length are very prominent. In the Poulsen arc the variations in frequency due to changes in the arc are so marked that the advantages of sharp tuning which undamped oscillations ought to give cannot be obtained in practice. On account of the general similarity of the characteristics of the Lepel arc to those of the Poulsen arc, it might be inferred that the frequency of the oscillations produced by the Lepel arc would vary in the same way as the frequency of the singing arc. In view of the importance of the question, however, it was thought desirable that the problem be made the subject of an experimental investigation.

§ 15. *The Frequency of the Duddell and Poulsen Oscillations.*

From theoretical considerations the author has derived[1] a formula for the frequency of the singing arc agreeing closely with all the experimental data which have been published on the Duddel and Poulsen arcs. In its general form this formula is

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{(R + dV/dA)^2}{4L^2}}, \qquad (1)$$

[1]Nasmyth, "The Frequency of the Singing Arc," PHYS. REV., *27*, No. 2, p. 117, August, 1908.

where n is the frequency, L is the inductance and C the capacity and R the resistance of the oscillatory circuit, and dV/dA is the slope of the volts-amperes characteristic curve. If we take the value for the slope as given by the author's experiments at high frequencies,

$$\frac{dV}{dA} = -\frac{c + ld}{A}, \tag{2}$$

where l is the arc length, A the arc current, and c and d are constants depending upon the electrodes and the atmosphere in which the arc is formed, the formula for the frequency becomes

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{\{R-(c+ld)/A\}^2}{4L^2}}. \tag{3}$$

If further, we neglect c and R, which are usually small in comparison with the term ld, we get the approximate formula,

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{l^2d^2}{4L^2A^2}}. \tag{4}$$

This formula shows that if the inductance is increased while the product LC is kept constant, the frequency will increase because the second term under the radical decreases. This effect was first observed experimentally by A. Banti.[1] If the arc current is increased the second term under the radical will decrease and the frequency should increase. Moreover, if the square of the frequency is plotted against the reciprocal of the square of the arc current a straight line with a slope $= -l^2d^2/4L^2$ should result, and the author has shown that this is the case both in his own experiments and in those of L. W. Austin.[2] Austin's data plotted in this way shows that the formula holds not only for the fundamental but for the harmonics which are present in the singing arc oscillations as well. The author has shown that the frequency decreases with increasing arc length and that if the square of the frequency is plotted against the square of the arc length a straight line results. Finally, the formula (3) shows that if the resistance of the oscillation circuit is

[1]A. Banti, Ellettricista, *12*, p. 1, January, 1903. See also L'Elettricita, March 8, 1908, p. 145.

[2]W. Austin, Bulletin, Bureau of Standards, *3*, p. 325, May, 1907.

increased the second term under the radical will approach zero, and the frequency should increase. This prediction has been verified by K. Vollmer,[1] who found that a resistance of 3.3 ohms added to the oscillatory circuit raised the frequency from 350,000 to 359,000 when the arc length was 0.3 mm. and the arc current 1.3 amperes, and similar, though smaller, increases of frequency on the addition of resistance at larger currents and arc lengths. It may be considered, therefore, that the law of the variation of the frequency of the singing arc is fairly well known, and may serve as a guide in the investigation of the frequency of the short-arc, metal-electrode generators.

§ 16. *The Frequency of the Short Arc Oscillations.*

The object of the first experiments on the frequency of the Lepel arc oscillations made by the author was to determine the dependence of the frequency on the arc current. A loosely coupled secondary with variable air condensers and a hot wire ammeter was used to determine the frequency, the connections being the same as in Fig. 1 in the section on the characteristics of short arcs.

Five Leyden jars having a total capacity of 0.0094 mfd. were used in the primary. Each of the three variable air condensers in the secondary had a maximum capacity of 0.00055 mfd., and differences of capacity of 0.000003 mfd. could be determined accurately by means of circular scales mounted on each of the air condensers and calibrated with considerable care by comparison with standard forms of capacity. The resistance of the primary circuit as measured with a Wheatstone bridge, was less than 0.10 ohm, and since stranded wires were used throughout, it is probable that the high frequency resistance did not greatly exceed this value.

The inductance of the primary circuit consisted of a solenoid of 15 turns, 20.55 cm. in length, and 12.85 cm. mean diameter. The stranded conductor was 0.65 cm. in diameter, and consisted of 36 strands each 0.6 mm. in diameter. The coefficient of self induction of this solenoid, as computed by Russell's formula,

$$L = (\pi dn)^2 l[1 - 0.424(d/l) + 0.125(d/l)^2 - 0.0156(d/l)^4],$$

[1] K. Vollmer, Jahrbuch der Drahtlosen Telegraphie und Telephonie, *3*, p. 143 (table 11), December, 1909.

where L is the inductance, d the diameter and l the length, in cm., and n is the number of turns per cm., was 14,100 cm. The value given by comparison with an Ayrton Perry standard at a frequency of 1,000 cycles was 14,400 cm. The small difference is probably due chiefly to the connecting wires at the end of the solenoid. Since these wires were used in the actual connections, the larger value given by direct measurement is probably the more correct. All the remaining connections in the primary were made so as to give a minimum of inductance, twisted pairs of wires being used wherever possible, and it is probable that the self inductance of the entire primary circuit was within one or two per cent. of 14,400 cm.

The inductance in the secondary circuit was a solenoid of 75 turns, 27.38 cm. long, and 8.87 cm. mean diameter. The computed coefficient of self induction was 140,200, and the measured value slightly greater than this, giving a mean of about 141,000 cm.

The primary and secondary circuits were coupled by arranging these two solenoids coaxially, with a distance of 7.2 cm. between their adjacent ends. In response to an inquiry concerning the best formula to use for the coefficient of mutual induction of the two solenoids, Dr. E. B. Rosa, of the Bureau of Standards, very kindly furnished a complete calculation of the coefficient by two formulas. According to an adaptation of the general formula given by Gray the mutual inductance was found to be 1,089.8 cm., while the formula of quadratures by Rayleigh gave 1,089.3 cm. The coefficient of mutual induction may be considered to be very closely $M = 1{,}090$ cm. therefore. Accordingly, the coupling coefficient is $k = M/\sqrt{L_1 L_2} = 1090/\sqrt{14{,}400 \times 141{,}000} = 0.0242$ or 2.42 per cent.

§ 17. *Variation with Arc Current.*

With the arc current maintained at a constant value, it was found that resonance could be obtained at a number of different frequencies by varying the capacity in the loosely coupled secondary circuit. It very soon became evident that the frequencies at which resonance could be obtained varied with the arc current. The results of the first experiments on the frequency are given later, in Fig. 17. As some of the resonance frequencies were close together, considerable difficulty was experienced in trying to follow the varia-

tions of a single one as the current was changed, and it was finally decided to make a complete exploration of the field. The results of the more exhaustive experiments at arc lengths of 0.10 and 0.15 mm. are given in Tables IV. and V., and graphically in Fig. 14.

TABLE IV.

Change of Frequency with Arc Current. (Fig. 14.)

Capacity. in M.F.	Frequency	Curve 1. Fundamental.		Curve 2a.		Curve 2.		Curve 2b.		Curve 3a.		Curve 3b.	
		D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.
.000545	565,000			2.81	.39	1.85	.39	1.42	.33	1.10	.28	.80	.25
.000697	500,000			2.10	.31	1.54	.44	1.22	.44			Curve 3.	
.000822	460,000			1.75	.36	1.40	.34	1.10	.33	.78	.41	.60	.41
.001091	400,000			1.75	.31	1.20	.40	.96	.41				
.001365	357,000			1.44	.30	1.06	.29	.87	.27				
.001635	327,000	2.59	.53	1.19	.23							Curve 1a.	
.001880	305,000	2.09	.29			.98	.14	.82	.12			2.58	.3
.002153	284,000	1.78	.22			.90	.12						
.002425	268,000	1.67	.15			.86	.10					Curve 1a.	
.00270	254,000	1.47	.15			.80	.07					1.85	.08
00297	242,000	1.30	.07										
.00325	232,000	1.23	.07										

D.C. indicates current through the arc, measured by a d.c. ammeter.

H.W. indicates intensity of oscillations, measured by a hot wire ammeter.

Arc. length, 0.10 millimeters, two sheets W. S. & B. Paragon paper.

Computed frequency from formula $n = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1}} = 430{,}000$.

Primary capacity 0.0094 mfd. Inductance 14,400 cm. (October 29, 1909).

It is evident that the frequency of the oscillations produced by the Lepel arc increases with the arc current, as in the singing arc. From the figure, the curves seem to occur in groups of three, and from the tables it is seen that the middle curves in each group, marked respectively 1, 2 and 3, represent the largest amounts of energy for a given current, the companion curves above and below representing comparatively weak oscillations. Curve 1 is apparently the fundamental, and at large currents of three amperes or more, this is the only resonance frequency that appears. It ap-

TABLE V.

Change of Frequency with Arc Current. (Fig. 14.)

Capacity in M.F.	Frequency.	Curve 1. Fundamental.		Curve 2a.		Curve 2.		Curve 2b.		Curve 3a.		Curve 3.	
		D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.
.000545	565,000	(Fig. 16)		2.70	.44	1.85	.33	1.42	.34	1.15	.33	.95	.33
.000697	500,000			2.36	.33	1.57	.42	1.26	.44	.85	.41	.70	.41
.000822	460,000			2.10	.44	1.45	.47	1.11	.48	.80	.50	.60	.44
.001091	400,000			1.76	.46	1.23	.41	.96	.44	.70	.42	Curve 3b.	
.001242	375,000			1.16	.42	1.18	.42					500,000	
.001365	357,000	3.47	.51	1.52	.38	1.06	.31					.60	.42
.001503	340,000	3.03	.50	1.43	.29								
.001635	327,000	2.58	.47	1.35	.23			.78	.17				
.001880	305,000	2.08	.30			1.05	.14			Curve 1a.		Curve 1b.	
.002153	284,000					.95	.14			2.43	.12	1.46	.21
.002425	268,000	1.76	.09			.89	.13			2.16	.13	1.34	.14
.00297	242,000	1.46	.07							1.85	.09		
.00352	223,000									1.63	.08		

D.C is current through the arc, measured by a d.c. ammeter.
H.W. is intensity of oscillations, measured by hot wire ammeter.
Arc length 0.15 millimeter, three sheets W. S. & B. Paragon paper.

Computed frequency from formula $n = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1}} = 430{,}000$.

Primary capacity = 0.0094 mfd. Inductance 14,400 cm. (November 11, 1909.)

proaches asymptotically the frequency given by the Thomson formula $n = I/2\pi\sqrt{L_1 C_1}$ as the arc current is increased. The curve 2 is evidently the first harmonic; at 2 amperes, for example, when curve 1 indicates a frequency of 300,000 for the fundamental, the frequency for curve 2 is very closely twice as great, 600,000. The range is not quite large enough to make sure that curve 3 is the next harmonic, but this is probably the case. At one ampere, for example, curve 2 indicates a frequency of about 300,000 and curve 3 about 600,000.

To avoid the necessity for the computation of a large number of frequencies corresponding to a varying capacity in the secondary, the capacity was held constant at the different values and the arc current varied by means of the resistance in the main circuit until

resonance was obtained. At higher frequencies than about 565,000 the oscillations in the secondary became so weak, on account of the small capacity, that accurate settings could not be obtained. The observations for the curves at 0.10 mm. were made before the

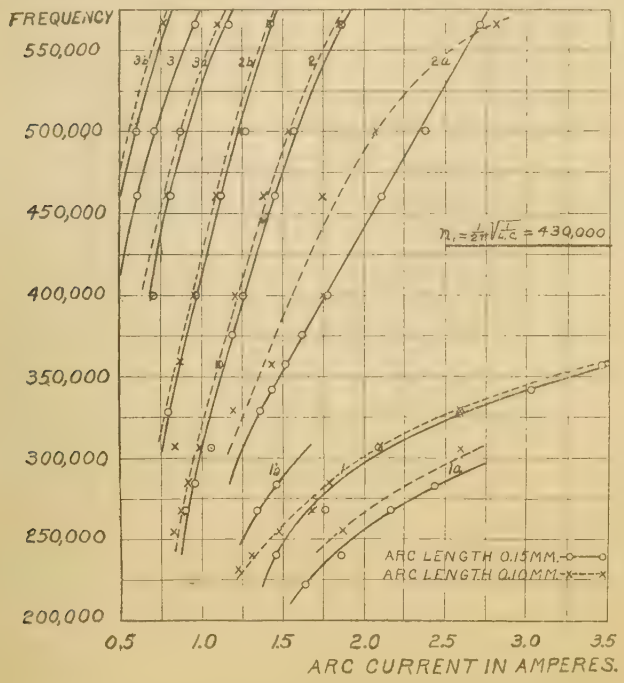

Fig. 14.
Change of frequency with arc current.

experiments at the longer arc length were undertaken, and the curves 1*b* and 3, which had been overlooked in the first observations were discovered in the more accurate experiments at 0.15 mm.

Influence of the Oscillatory Current.—The interpretation of the apparent discrepancies in the two sets of curves is interesting. The reason for the flatness of the curve 2*a* at 0.15 mm. in the region from about 1.5 to 2.5 amperes is clear from Fig. 7 in the section on the characteristics. Fig. 7 shows that the region of maximum voltage

at 0.15 mm. is between 1.5 and 2.5 amperes, and Fig. 8 shows that the region of minimum oscillations corresponds with the region of maximum voltage. The currents plotted in Fig. 14 were measured by a d.c. ammeter in the main circuit, whereas the actual current which determines the resistance and characteristics of the arc includes not only this current but the oscillatory current as well. The effective current traversing the arc is accordingly less, relatively in the region where the full line curve *2a* is flat, and if total currents through the arc were plotted instead of the d.c. current, the flat part of the curve would bend over into its proper place in the region of smaller currents. The apparent shift of the curve *1a*, and of most of the points in curve 1, in the same region, is explained in the same way. At 0.10 mm. the region of high voltages and low oscillation currents moves to higher currents as shown in Fig. 7, and the effect of this is apparent in the falling off of the dotted curve *2a* at 2.81 amperes d.c. No readings at higher currents than 2.59 amperes were obtained at 0.10 mm. and the dotted curve has been drawn as if it lay above the full line curve all the way out, but the two curves probably cross at larger currents, on account of the influence of the minimum in the oscillatory current. At small currents no irregularities are observed, as no maxima of voltages or minima of oscillations occur in this region.

If the effective currents through the arc were plotted instead of the readings of the d.c. ammeter, the curves at low currents would be affected more than the others, since the oscillatory currents are large even when the d.c. ammeter readings are small. For the same values of effective current, the frequencies indicated by all the curves would be less, but the frequencies at small d.c. readings would be reduced more than the others, and the curves would be steeper.

Influence of Mutual Inductance.—Although the coupling between the primary and the secondary resonance circuit was very loose, the coefficient being only 0.0242 or 2.42 per cent., it was thought possible that the two companion curves above and below the curves 1, 2 and 3 might be accounted for as the two coupled circuit waves which appear even in impact excitation. It is difficult to see how indications of the two coupled waves could be obtained without the

aid of a loosely coupled tertiary circuit, and moreover the largest effect of the loose coupling employed turns out to be far too small to account for the phenomena. At resonance the frequencies[1] of the two waves of the coupled circuit should be

$$T = \sqrt{T_0^2 \pm \theta^2}; \quad (\theta^2 = 4\pi^2 M \sqrt{C_1 C_2})$$

where T_0 is the period of each circuit before coupling, M is the coefficient of mutual induction between the two circuits, and C_1 and C_2 are the capacities in the primary and secondary circuits respectively. Using the actual capacity, 0.0094 mfd. in the primary and the theoretical capacity from the Thomson formula, 0.00092 mfd. in the secondary, we get for the two frequencies 425,000 and 435,000 respectively. This gives a difference of only 1.16 per cent. instead of the difference of more than 10 per cent. required to account for the observations. Using the actual values of the secondary capacity, for a frequency of 350,000 the coupled circuit frequencies would be 347,800 and 352,200, giving a still smaller difference of less than three fourths of one per cent. The phenomenon seems to be therefore a characteristic of the generator itself rather than of the coupling. The presence of the large number of harmonics at which resonance can be obtained with the Lepel arc marks another point of resemblance between it and the singing arc. Austin[2] found three frequencies at which resonance could be obtained, all of which increased with the arc current. In the author's experiments on the frequency of the singing arc formed between a copper and a carbon electrode in illuminating gas six such frequencies were found by means of a loosely coupled resonance circuit, with indications of others at capacities too small to measure accurately.

§ 18. *Variation with Arc Length.*

The full line and dotted line curves for arc lengths of 0.15 and 0.10 mm. given in Fig. 14 show that the frequency decreases with increasing arc length, as in the case of the singing arc and in agreement with the formula (3) or (4). On attempting to follow the changes in the curve 1 as the arc length was increased apparent

[1] See "The Principles of Electric Wave Telegraphy," J. A. Fleming.
[2] Austin, *loc. cit.* See also PHYS. REV., 30, p. 134, August, 1908.

inconsistencies were again met with, the curves falling below each other for a time, then rising to higher frequencies as the arc length was increased beyond a critical value and finally falling again. The attempt to trace the course of the whole curve 1 was finally abandoned, and the variation of the frequency of the fundamental with

Table VI.

Fig. 15. Variation of Frequency with Arc Length.

Sheets Paper.	2	3	4	6	8	10	12	14
Arc length, mm.	0.10	0.15	0.20	0.30	0.40	0.50	0.60	0.70
Arc length squared	0.01	0.023	0.04	0.09	0.16	0.25	0.36	0.49
Fundamental at 2.0 amperes. C_1=0.0094 mfd.								
Frequency $\div 10^3$	303	299	315	305	295	288	275	—
(Frequency)$^2 \div 10^{10}$	9.18	8.94	9.92	9.40	8.70	8.29	7.56	—
Harmonic at 1.0 ampere. C_1=0.00564 mfd.								
Frequency $\div 10^3$	1,095	—	999	885	910	850	715	662
(Frequency)$^2 \div 10^{10}$	119.9	—	99.8	78.4	82.8	72.3	51.1	43.8

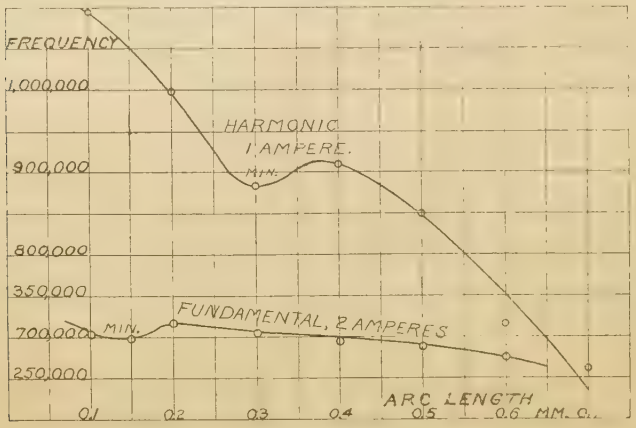

Fig. 15.
Variation of Frequency with Arc Length.

arc length at a constant current of 2 amperes was investigated. The results obtained are given in Table VI. and graphically in Fig. 15. With a primary capacity of 0.00564 mfd. and the same primary

inductance of 14,400 cm., the variation of frequency of one of the harmonics at 1.0 ampere was also observed and the corresponding curve, which is of the same general form, has been plotted. The values of the squares of the arc lengths and frequencies are also given for use later in § 19.

Influence of the Oscillatory Current.—In both the curves of Fig. 15 the frequency decreases with increasing arc length except for regions of strongly marked minima in each case. On referring to

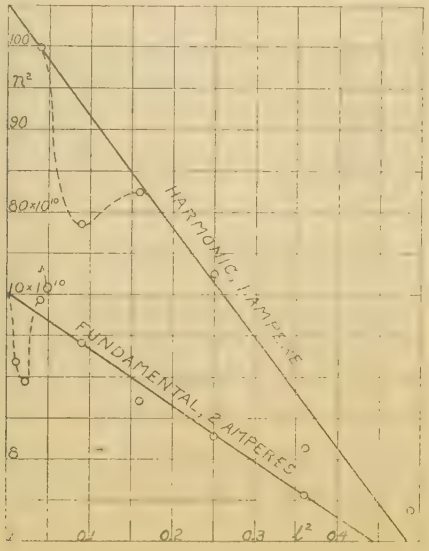

Fig. 16.

Squares of Frequency and Arc Length.

Physical Equation of Straight Lines, $n^2 = -\frac{d^2}{16\pi^2 L^2 A^2} \cdot l^2 + \frac{1}{4\pi^2 LC}$.

Fig. 7 again, it is found that these minima correspond to regions of voltage maxima and oscillatory current minima, so that the reason for the apparent discrepancies are traced as before, to a decrease in the effective current through the arc. Thus, the minimum in the oscillations and in the frequency for the 2 amperes d.c. curve both come at 0.15 mm. For the harmonic a smaller capacity and a

smaller current were used than for the curves in Fig. 7, and the minimum should be shifted into the region of longer arc lengths or smaller capacities, as it is. If this allowance is made, we may say that the minima in the oscillations and the frequency both come at about 0.3 mm. A second minimum, less pronounced, seems to be indicated at about 0.6 mm. in the curve for the frequency, and it is possible that the corresponding voltage maximum lies just outside the range of Fig. 7.

§ 19. *Proof of the Formula for the Frequency.*

It is apparent from the above curves that the frequency of the Lepel arc agrees qualitatively with the formula which has been found to hold true for the singing arc. With the exception of the

TABLE VII.

Figs. 17 and 18. D.C. Arc. Frequency and Arc Current.

Current $=I$.	$1/I^2$.	Frequency $=n$.	n^2	
		Curve 1.		
3.47	0.083	357,000	12.74×10^{10}	Copper electrodes.
3.03	0.109	340,000	11.56	Arc length $=0.15$ mm.
2.58	0.150	327,000	10.69	$L_1 = 14.400$ cm.
2.08	0.230	305,000	9.30	$C_1 = 0.0094$ mfd.
1.76	0.320	268,000	7.18	$N_0 = 430{,}000$.
1.46	0.460	242,000	5.86	
		Curve 2.		
1.18	0.720	322,000	10.36×10^{10}	
1.29	0.600	332,000	11.02	
1.38	0.530	335,000	11.22	
1.43	0.490	337,000	11.36	
1.48	0.456	342,000	11.70	
1.52	0.430	347,000	12.04	
1.55	0.417	352,000	12.39	Copper+, brass−, electrodes
1.595	0.393	357,000	12.76	Arc length $=0.35$ mm.
1.66	0.362	362,000	13.10	$L_1 = 14{,}400$ cm.
1.70	0.346	369,000	13.60	$C_1 = 0.00564$ mfd.
1.79	0.312	366,000	13.40	$M = 1089$ cm.
1.83	0.299	370,000	13.70	
1.87	0.286	379,000	14.30	
1.19	0.705	329,000	10.82	
1.08	0.858	315,000	9.91	
.98	1.040	298,000	8.90	
.875	1.310	275,000	7.58	

Curve 3.

2.34	0.183	711,000	50.7×10^{10}	
2.19	0.209	694,000	48.1	
2.06	0.235	678,000	46.0	Copper+, brass−, electrodes.
1.93	0.269	668,000	44.6	Arc length 0.15 mm.
1.79	0.312	650,000	42.3	$L_1 = 14{,}400$ cm.
1.61	0.389	625,000	39.1	$C_1 = \begin{cases} 0.00376 \\ 0.00564 \end{cases}$
1.40	0.510	571,000	32.6	
1.30	0.590	534,000	28.5	

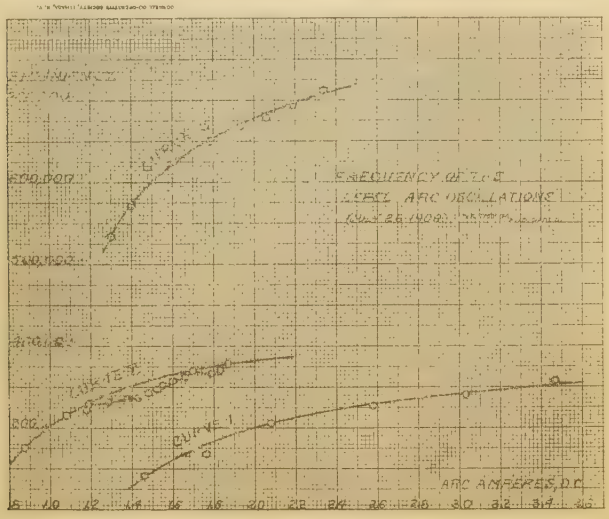

Fig. 17.

Frequency and arc current.

irregularities due to the oscillation minima the curves are of the same form as the corresponding singing arc curves. A more rigorous test of the formula is obtained by plotting the square of the frequency against the square of the arc length, as in Fig. 16. Both these curves prove to be straight lines outside the regions of the above mentioned irregularities due to a decrease in the intensity of the oscillatory current through the arc. If the effective current through the arc A, as measured by a hot wire ammeter, had been maintained constant instead of the direct current I, the agreement

of the curves with the straight line requirement of the formula would have been as satisfactory as in the case of the Poulsen arc.

Additional data for the variation of frequency with the arc current are given in Table VII., and the curves corresponding are plotted in Fig. 17. The data were obtained with the same connections and coupling as for the curves in Fig. 14. The effect of the minima in the oscillations is plainly visible in curve 2, and in the corresponding straight line in Fig. 18. The data for curve 2 are

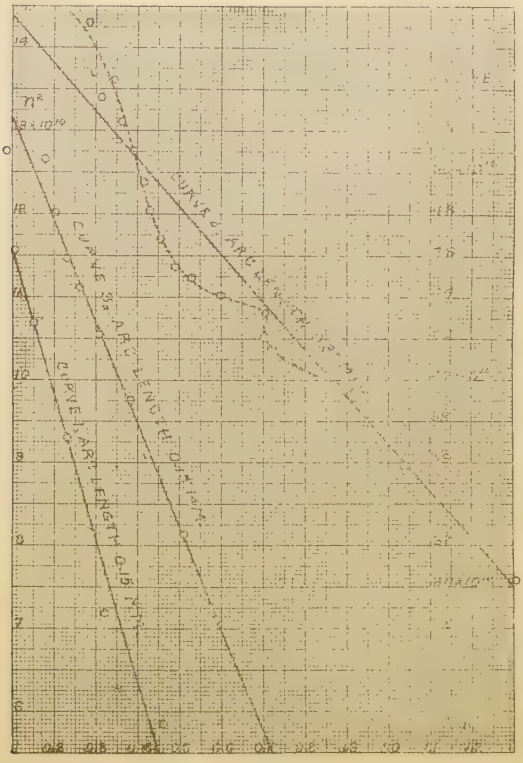

Fig. 18.

Squares of frequency and current reciprocals.

Physical equation of straight lines, $n^2 = -\frac{l^2 d^2}{16\pi^2 L^2} \cdot \frac{1}{A^2} + \frac{1}{4\pi^2 LC}$.

given in Table VII. in the same order in which they were obtained, and it is apparent that the points taken near the end of the rim form a much smoother curve than the earlier ones. It is possible that in this case there was some irregularity in the arc which caused the minima in the oscillations, in addition to the causes which have been traced in the preceding sections on the intensity of the oscillations.

With the exception of the discrepancy caused by using the arc current as measured by a d.c. ammeter instead of the effective current in curve 2, the agreement with the straight line requirement of the formula is again well within the limits of errors of observation.

§ 20. *Related Experiments on Poulsen Oscillations.*

The experiments of the author have shown that the Lepel arc is closely related to the singing arc, especially in its frequency characteristics. Since it appears that the formula derived by the author for the singing arc holds also for the short arc between metal electrodes, experiments on the singing arc which afford additional tests for this formula are of interest to investigators working with the new impact generators.

Some excellent experimental data recently obtained by K. Vollmer[1] working under the direction of Professor Max Wien at Danzig, afford material for a severe quantitative test of the formula as applied to Poulsen oscillations. Vollmer used flat discs of copper and carbon as his electrodes, and formed the arc in an atmosphere of hydrogen. His experiments cover a wide range of frequencies, from 156,000 to 1,000,000 per second corresponding to wave-lengths of from 1,915 to 300 meters. The inductances used range from 33,300 cm. to 1,358,000 cm. and the capacities from 0.00076 mfd. to 0.00585 mfd.

Vollmer measured his arc lengths directly, and did not take account of the variations in the length as indicated by the changes in voltage. Nevertheless the two curves[2] which he plotted to verify the author's formula for the frequency give a good straight line between n^2 and l^2 as formula (4) requires when l is large enough so

[1] Vollmer, Jahrbuch der Drahtlosen Telegraphie und Telegraphonie, *3*, pp. 117–173.
[2] Vollmer, *loc. cit.*, p. 146.

Variation of Frequency with Arc Length; Poulsen Arc.

Table 2V.

$n_0 = 156{,}600$; $L_1 = 1{,}358{,}000$ *cm.*; $C_1 = 0.76 \times 10^{-3}$ *mf.*; $\lambda_0 = 1915$ *meters.*
(I_g = *generator or arc current as measured by a d.c. ammeter;* I_c = *oscillatory current.*)
$I_g = 2.7$ *amp.* $I_c = 2.0$ *amp.*

l	l^2	n^2	
1.2 mm.	1.44	2.433×10^{10}	$m = 11 \times 10^6$
1.8	3.24	2.4313	$\sqrt{m} = 3310$
2.4	5.76	2.4298	$d = 154$
2.7	7.29	2.4267	
3.3	10.89	2.4250	$d\sqrt{C_1} = 4.25$
3.6	12.96	2.4212	

Table 3V.

$n_0 = 156{,}600$; $L_1 = 532{,}000$ *cm.*; $C_1 = 1.94 \times 10^{-3}$ *mf;* $I_g = 2.7$ *amperes;* $I_c = 2.01$ *amp.*

l	l^2	n^2	
0.3 mm.	0.09	2.4414×10^{10}	
0.6	0.36	2.4388	
0.9	0.81	2.4363	$m = 27 \times 10^6$
1.2	1.44	2.4348	$\sqrt{m} = 5200$
1.8	3.24	2.4280	$d = 94$
2.1	4.41	2.4267	
2.4	5.76	2.4230	$d\sqrt{C_1} = 4.14$
2.7	7.29	2.4200	
3.0	9.00	2.4154	
3.3	10.89	2.4154	

Table 4V.

$n_0 = 156{,}600$; $L_1 = 176{,}500$ *cm.*; $C_1 = 5.85 \times 10^{-3}$ *mf.*; $I_g = 2.7$ *amp.*; $I_c = 2.1$ *amp.*

l	l^2	n^2	
0.3 mm.	0.09	2.434×10^{10}	$m = 92 \times 10^6$
0.6	0.36	2.421	$\sqrt{m} = 9{,}600$ (9,600)
0.9	0.81	2.419	$d = 57.5$
1.2	1.44	2.416	$d\sqrt{C_1} = 4.40$
1.5	2.25	2.396	
1.8	3.24	2.3815	
2.1	4.41	2.374	
2.4	5.76	2.357	
3.0	9.00	2.343	
3.6	12.96	2.326	
3.9	15.21	2.282	

TABLE 5V.

$n_0 = 375{,}000$; $L_1 = 237{,}000$ *cm.*; $C_1 = 0.76 \times 10^{-3}$ *mf.*; $\lambda_0 = 800$ *meters*; $I_g = 2.7$ *amp.*; $I_c' = 2.15$ *amp.*

l	l^2	n^2	
1.1	1.21	13.684×10^{10}	$m = 5 \times 10^8$
1.2	1.44	13.65	$\sqrt{m} = 2.24 \times 10^4$
1.35	1.82	13.624	$d = 180$
1.5	2.25	13.583	$d\sqrt{C_1} = 4.96$
1.8	3.24	13.55	

TABLE 6V.

$n_0 = 375{,}000$; $L_1 = 92{,}900$ *cm.*; $C_1 = 1.94 \times 10^{-3}$ *mf.*; $I_g = 2.7$ *amp.*; $I_c = 2.15$ *amp.*

l	l^2	n^2	
0.3 mm.	0.09	13.86×10^{10}	
0.45	0.203	13.80	$m = 10 \times 10^8$
0.6	0.36	13.73	$\sqrt{m} = 3.16 \times 10^4$
0.9	0.81	13.68	$d = 100$
1.2	1.44	13.63	
1.5	2.25	13.51	$d\sqrt{C_1} = 4.40$
1.8	3.24	13.46	
2.1	4.41	13.35	
2.4	5.78	13.13	

TABLE 7V.

$n_0 = 375{,}000$; $L_1 = 30{,}800$ *cm.*; $C_1 = 5.85 \times 10^{-3}$ *mf.*; $I_g = 2.7$ *amp.*; $I_c = 2.25$ *amp.*

l	l^2	n^2	
0.15 mm.	0 0225	13.8×10^{10}	
0.3	0.09	13.4	$m = 33 \times 10^8$
0.6	0 36	13.1	$\sqrt{m} = 5.75 \times 10^4$
0.75	0.5625	12.8	$d = 60$
0.9	0.81	12.6	
1.2	1.44	12.32	$d\sqrt{C_1} = 4.59$
1.5	2.25	12.1	
1.8	3.24	11.7	
2.1	4.41	11.3	
2.4	5.76	11.0	

that the constant c may be neglected. The Thomson frequencies chosen for the two tests were 156,600 with a capacity of 0.00585 mfd. and an inductance of 176,500 cm.; and 375,000, with a capacity of 0.00076 mfd. and an inductance of 237,000 cm. Although the

TABLE 8V.

$n_0 = 1{,}000{,}000$; $L_1 = 33{,}300$ *cm.*; $C_1 = 0.76 \times 10^{-3}$ *mf.*; $\lambda_0 = 300$ *meters*; $I_g = 2.7$ *amp.*; $I_c = 1.7$ *amp.*

l	l^2	n^2	
1.2 mm.	1.44	92.2×10^{10}	$m = 248 \times 10^8$
1.4	1.96	91.0	$\sqrt{m} = 15.7 \times 10^4$
1.5	2.25	89.7	$d = 177$
1.7	2.89	88.7	$d\sqrt{C_1} = 4.89$

TABLE 9V.

$n_0 = 1{,}000{,}000$; $I_1 = 13{,}060$ *cm.*; $C_1 = 1.94 \times 10^{-3}$ *mf.*; $I_g = 2.7$ *amp.*; $I_c = 2.1$ *amp.*

l	l^2	n^2	
0.3 mm.	0.09	91.4×10^{10}	$m = 436 \times 10^8$
0.45	0.2025	90.2	$\sqrt{m} = 20.9 \times 10^4$
0.9	0.81	86.4	$d = 93$
1.2	1.44	84.2	
1.5	2.25	81.2	$d\sqrt{C_1} = 4.10$
1.7	2.89	78.8	

Recapitulation.

No.	L_1	C_1	I_c	$\sqrt{m}$	d	$d\sqrt{C_1}$
156,600	1,358,000	0.76×10^{-3}	2.0	0.33×10^4	154	4.25
156,600	532,000	1.94	2.01	0.52	94	4.14
156,600	176,500	5.85	2.1	0.96	57.5	4.40
375,000	237,000	0.76	2.15	2.24×10^4	180	4.96
375,000	92,900	1.94	2.15	3.16	100	4.40
375,000	30,800	5.75	2.25	5.75	60	4.59
1,000,000	33,300	0.76	1.7	15.7×10^4	177	4.89
1,000,000	13,060	1.94	2.1	20.9	93	4.10

$I_g = 2.70$ amperes. Mean 4.47

curves given are straight lines, as required by the formula, the value of the constant d as found from the slope is greater for the large capacity and low frequency curve than for the other. The author found that the product of the constant d and the square root of capacity gives a constant for these cases. Testing Vollmer's other remaining data in the same way, it was found that the product of $d\sqrt{C_1}$ gave a constant within the limits of the errors of observation, with a mean value of 4.47, in all his experiments. Vollmer's data

are not in convenient form for testing the formula, as he gives wave-lengths instead of frequencies. The data are therefore given in the form for direct verification of the results in tables 2V. to 9V., corresponding to Vollmer's table numbers, 2 to 9.

It was predicted in the derivation of the formula[1] that the constant d would change with the Thomson frequency on account of the skin effect of resistance in the arc vapor. It would also change with the capacity in parallel with the arc on account of the effect of capacity on the arc voltage-current characteristic. It appears from the above results, however, that if we use for d the value $d = k/\sqrt{C_1}$, where $k = 4.47 \pm 0.4$, the formula may be extended from a single Thomson frequency (constant value of L_1 and C_1) for which it was at first derived, to include all of Vollmer's experiments from 156,600 to 1,000,000 frequency range and with all values of capacity and inductance used in the practice of radio-telegraphy and radio-telephony.

Using the above value of d, the approximate formula (4) becomes

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{l^2k^2}{4L^2CA^2}}, \qquad (5)$$

where n is the frequency, L and C the inductance and capacity, A the arc current, l the arc length, and k is a constant for given arc electrodes and atmosphere. The agreement of the generalized formula (5) with the results of experiments may be shown graphically if we plot n^2L^2C against l^2. The equation of the straight line in the form $y = mx + b$ is

$$n^2L^2C = -\frac{k^2}{16\pi^2A^2}l^2 + \frac{L}{4\pi}.$$

For a constant current A all the lines should have the same slope, $-k^2/16\pi^2A^2$. The results from Vollmer's data are given in convenient form in Table X., and the curves at constant slope are plotted in Fig. 17.

The agreement is seen to be well within the limits of the errors of observation in Vollmer's work.

Vollmer measured his arc currents with a d.c. ammeter, whereas

[1] PHYS. REV., 27, p. 127, formula 13.

the author's experiments on the variation of the frequency of the Lepel arc with current and arc length have demonstrated that the effective current through the arc, including the oscillations, should be used as the arc current A in the formula. The oscillatory current in general increases with the capacity, so that the correction on this account which should be made in Vollmer's data is in the right direction. Whether the Thomson frequency and the oscillatory current corrections together are sufficient to account for the constancy of the product $d\sqrt{C_1}$ must be determined by further experiments in which the arc current is measured by a hot wire ammeter or some equivalent instrument. The relation is at any rate interesting, and enables us to extend the formula from a single frequency as determined by the capacity and inductance, to the general case.

Vollmer's data show also, as mentioned above, that the frequency increases with the resistance in the oscillation circuit. This is in agreement with the prediction of formula (3) and contradicts all

Table X.

(*Figure* 19.) *Frequency and Arc Length; Poulsen Arc. Values of* n^2L^2C.

l^2	Table 2V.	Table 3V.	Table 4V.	Table 5V.	Table 6V.	Table 7V.	Table 8V.	Table 9V.
.0225						.766		
.090		13.403	4.437		2.176	.744		.303
.203					2.167			.299
.360		13.389	4.413		2.156	.727		
.5625						.7104		
.81		13.375	4.410		2.148	.699		.286
1.21				5.841				
1.44	34.11	13.367	4.404	5.829	2.140	.684	.777	.279
1.82				5.817				
1.96							.767	
2.25			4.368	5.801	2.121	.672	.756	.269
2.89							.748	.261
3.24	34.08	13.330	4.341	5.786	2.113	.6494		
4.41		13.323	4.328		2.096	.627		
5.76	34.06	13.302	4.297		2.061	.611		
7.29	34.01	13.286						
9.00		13.261	4.271					
10.89	33 99	13.261						
12.96	33.94		4.240					
15.21			4.160					

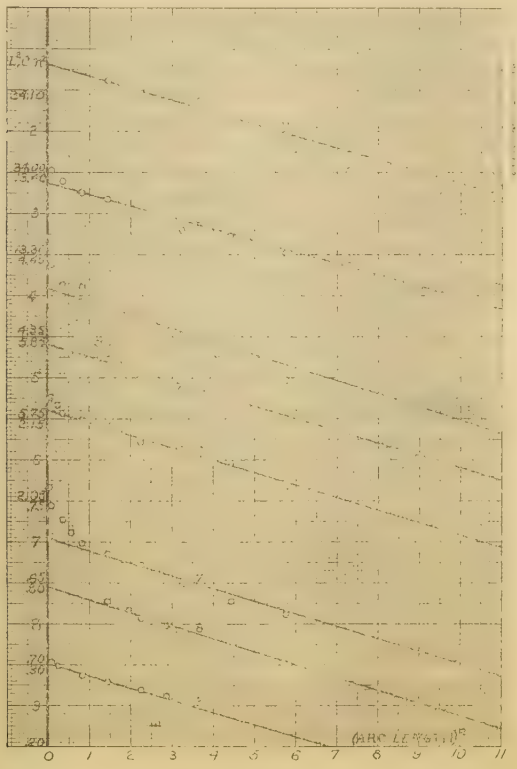

Fig. 19.

Frequency arc length curves with constant slope.

previous assumptions of investigators, based for the most part on reasoning by analogy from the effect of friction in mechanical pendulums and other systems. There is reason to suspect that an increase of frequency with resistance will be found also in the ordinary spark gap excitation as well as in the short spark generators.

§ 21. *Impact Excitation and Forced Undamped Oscillations.*

The resonance curve for the antenna or secondary of the two closely coupled circuits is of interest on account of its bearing upon

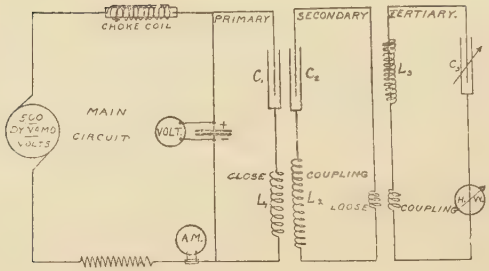

Fig. 20.

Diagram of connections for resonance curves.

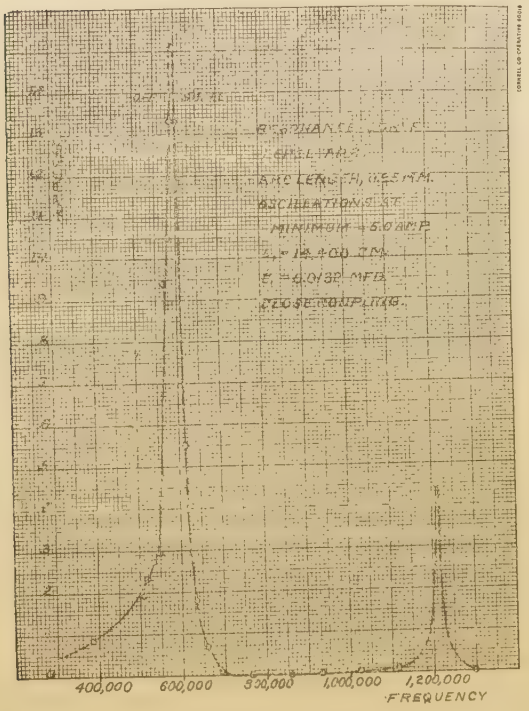

Fig. 21.

Resonance curve for Lepel arc.

the question of impact excitation. It may be determined by means of a tertiary circuit in addition to the connections used thus far in the experiments and shown in Fig. 1, loosely coupled to the

TABLE XI.

Resonance Curve for Lepel Arc.

Frequency.	Hot Wire Ammeter Reading.	
385,000	.26	Primary capacity, 7 ja s = 0.0132 mfd.
385,000	.40	Primary inductance, 14,400 cm.
399,000	.40	Arc length, 0.15 mm. Volts, 80.
415,000	.46	Arc current, 2.15 amperes d.c.
420,000	.48	Oscillations at minimum, 4.6 amperes.
440,000	.46	Secondary capacity, 2 jars in series = 0.00094 mfd.
450,000	.43	Secondary inductance, about 180,000 cm.
470,000	.48	
482,000	.52	
492,000	.625	
502,000	.725	Secondary and primary closely coupled. Secondary
506,000	.83	solenoid of 141,000 cm. inductance placed inside
519,000	.94	primary solenoid of 14,100 cm. inductance with
520,000	1.166	lower ends flush. See § 16.
525,000	1.37	
Maximum, off scale.		
567,000	1.37	Tertiary capacity variable.
579,000	.92	Tertiary inductance 160,000.
594,000	.79	Tertiary and secondary loosely coupled.
596,000	.68	
615,000	.40	
660,000	.26	For the resonance curve of Figure 21 the primary and
702,000	.185	secondary capacities and inductance were the same,
765,000	.15	but the arc length was 0.25 mm., and the arc current
980,000	.28	3.03 amperes (volts 66) giving a minimum of 5.0
1,060,000	.51	amperes in the oscillations.
Maximum. First harmonic.		
1,000,000	.29	
1,150,000	.20	
1,220,000	.10	

secondary. The complete diagram of connections for determining the form of the wave in the antenna or secondary is given in Fig. 20. The primary and secondary circuits are maintained constant, and

the readings of the hot wire ammeter are read as the capacity in the loosely coupled tertiary circuit is varied.

With the arc current arranged so as to give a minimum in the intensity of the oscillations at an arc length of 0.25 mm. the resonance curve shown in Fig. 21 was obtained.

The primary and secondary circuits were not tuned to the same frequency, the lowering of the frequency of the oscillations by the arc itself increasing the so-called "untuning." The arc current was 3.03 amperes d.c., and the voltage 66. The remaining constants are given in connection with Table XI., which gives the data for a similar resonance curve (not plotted) at an arc length of 0.15 mm. and with the oscillations again at a minimum.

Both the resonance curves show but a single wave, the smaller maximum being evidently the harmonic of double frequency. So far they are in agreement with the theory of impact excitation, the steep curves and sharp peak being especially characteristic. On the other hand, the frequency of the oscillations should remain constant, as long as the secondary circuit is not changed, whereas the frequency of the oscillations changes with the constants of the primary circuit, according to the above figure and table. Further experiments gave the following increase of frequency of the oscillations in the secondary circuit with increasing primary arc current.

Table XII.

Arc Current Amperes, D.C.	Arc Volts, D.C.	Oscillation Amperes.	Resonance Frequency.
1.30	90	3.25	500,000
1.50	88	3.40	507,000
1.75	84	3.45	519,000
2.00	90	4.27	528,000
2.00	80	3.68	545,000
2.00	70	3.68	552,000
3.00	66	4.84	563,000

The conclusion is that in these experiments at least, the Lepel arc did not give free impact excitation, but *forced oscillations* which the form of the resonance curve shows to be almost *undamped*

at arc currents which give a minimum in the intensity of the oscillations.

Summary.

The Lepel generator resembles the older Duddell and Poulsen arc generators in having a number of harmonics, besides the fundamental, to which a loosely coupled resonance circuit may be tuned.

The following laws hold for both fundamental and harmonics, as in the case of the older generators:

1. The frequency of the oscillations increases with increasing arc current at constant arc length.

2. The frequency of the oscillations increases with decreasing arc length at constant arc current.

The variation in the frequency is least with a large primary inductance.

It is predicted, though not yet experimentally verified, that a small increase in the resistance of the oscillatory circuit will increase the frequency of the Lepel arc oscillations as in the case of the Duddell and Poulsen arcs.

The formula derived by the author in 1908 for the frequency of the singing arc, which includes all the known facts and is in agreement with the results of all the experiments on the frequency of the Duddell and Poulsen arc oscillations, holds also for the Lepel arc oscillations. The formula is

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{\{R-(c+ld)/A\}^2}{4L^2}},$$

where n is the frequency, L, C and R the inductance, capacity and resistance of the condensor circuit, l the arc length and A the arc current, and c and d are constants depending upon the electrodes and gas in which the arc is formed. Usually R and c are small compared with the other terms, so that the formula may be written in an approximate form,

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{l^2d^2}{4L^2A^2}}.$$

Recent extensive experiments on the Poulsen arc have shown that if the value of $d = k/\sqrt{C}$ where d is the arc constant and C

the primary capacity, k has the same value for a given arc for all the values of inductance, capacity and Thomson frequencies used in practice. In its more general form the formula thus becomes

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{l^2k^2}{4L^2A^2C}}.$$

The effect of capacity on the oscillation intensity and the form of the voltage current characteristics makes it very probable that the more generalized formula holds for the Lepel arc also, but sufficient data for a rigorous test are not yet available.

With a closely coupled secondary circuit and the arc current arranged to give minimum oscillations, a very sharp resonance curve with only a single wave is obtained by means of a loosely coupled tertiary circuit, as required by the theory of impact excitation. The maximum shifts with change of arc current, however, leading to the conclusion that the sharp resonance curve of the Lepel arc is due, not to free oscillations, but to *forced undamped* oscillations.

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